

00348

**M.Sc. (MATHEMATICS WITH  
APPLICATIONS IN COMPUTER SCIENCE)  
M.Sc. (MACS)**

**Term-End Examination**

**June, 2011**

**MMT-005 : COMPLEX ANALYSIS**

*Time : 1½ hours*

*Maximum Marks : 25*

*Note : Question No. 1 is compulsory . Attempt any three other questions. Use of calculator is **not** allowed.*

1. State giving reasons whether the following statements are *true* or *false*. **5x2=10**

(a)  $f(z) = \sin\left(\frac{1}{z^2}\right)$  has only one singularity

which is a pole of order 2 at  $z=0$ .

(b)  $f(z) = \bar{z}$  is continuous in the whole complex plane but is nowhere differentiable.

(c)  $f(z) = \tan z$  is an entire function.

(d) If  $\int_C f(z) dz = 0$  for a function  $f(z)$  where  $C$

is any simple closed contour in  $\mathbb{C}$  domain  $D$  then  $f(z)$  is analytic in  $D$ .

(e) If  $f(z) = \frac{1}{z^4 - 4z^2 + 3}$  then the maximum

value of  $f(z)$  is attained at  $z=2$ .

2. (a) Find the bilinear transformation which takes the points  $1, 0, \infty$  to  $-1, i, -i$ , 2  
 (b) Using  $\epsilon - \delta$  definition of limit prove that 3

$$\lim_{z \rightarrow 1} (z^2) = 1.$$

3. (a) Find an analytic function whose real part is the function  $4(x, y) = 3x + y$ . 2  
 (b) Find all the roots of the equation  $\sinh z = i$ . 3

4. (a) Let  $c$  be the circle  $|z| = 4$ , described in the positive sense. 3

$$\text{If } g(w) = \int_C \frac{2z^2 + z + 2}{(z-w)} dz, \quad w \notin c, \text{ then}$$

find  $g(2)$ . What is the value of  $g(w)$  for  $|w| > 4$ ?

- (b) If  $f(z)$  is an entire function such that  $|f(z)| \leq 2|z|$  for all  $z$ , then show that  $f(z) = Az$ , where  $A$  is a complex coefficient. 2

5. Show that  $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ . 5