No. of Printed Pages : 4 **MMT-004**

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

June, 2011

MMT-004 : REAL ANALYSIS

Time : 2 hours *Maximum Marks : 50*

Note : *Question no.* **1** *is* ***compulsory****. Do* ***any four*** *questions out of question nos.* **2** *to* **7***. Calculators are* ***not*** *allowed.*

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1. State, whether the following statements are True or False. Give reasons for your answer : **5x2=10**

(a) [0, 2] is a connected set in **R** with discrete metric.

(b) The Sequence $\left\{\left(\frac{1}{n}, \frac{1}{n}\right) : n \in N\right\}$ is convergent in $\mathbf{R}^2$ with discrete metric.

(c) Let $E_n = \left[0, \frac{1}{n}\right]$, $n \in N$. Then $m\,(\bigcap E_n) = 0$.

(d) $\int_{\mathbf{R}} \chi_Q \, dm = 0$, where Q is the set of rational numbers.

(e) (0, 1, -1) is a critical point of the function $f(x, y, z) = 1 + |x| + |y| + |z|$

2. (a) Define the outer measure m* of a set $A \subseteq \mathbf{R}$ 3

Find the outer measure of the following sets.

(i) $A = [3, 4] \cup \{x : x$ is a solution of the equation $x^2 + 1 = 0\}$

(ii) $A = \{ r : r$ is a rational number in $[0, 1]\}$

(b) Check whether the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by $f(x, y) = 2x^4 - 3x^2y + y^2$ has local minima. 3

(c) Let (X, d) and (Y, d') be two metric spaces and f be a continuous function from X to Y, and $x_0 \in X$. Show that f is continuous if and only if for every sequence $\{x_n\}$ in X converging to x_0, $f(x_n)$ converges to $f(x_0)$. 4

3. (a) Prove that every compact set in a metric space is closed and bounded. Is the converse true ? Justify. 6

(b) Find the directional derivative of the function $f : \mathbf{R}^3 \to \mathbf{R}$ given by $f(x, y, z) = y^2 + 2xz$ in the direction $V = (0, 1, 2)$ at the point $(1, 2, -3)$. 4

4. (a) If E_1 and E_2 are measurable sets and $E_1 \cap E_2 = \phi$, then prove that $E_1 \cup E_2$ is measurable. 5

(b) Check the differentiability of the following functions at the indicated points. Find the derivative wherever they exist. 5

(i) The function $f : \mathbf{R}^2 \to \mathbf{R}$ given by $f(x_1, x_2) = |x_1| + |x_2|$, $a = (1, 0)$.

(ii) The function $f : \mathbf{R}^4 \to \mathbf{R}^2$ given by : $f(x_1, x_2, x_3, x_4) = (x_1^2 - x_2^2, x_3^2 - x_4^2)$ at $a = (1, 0, 2, -1)$.

5. (a) Define Fourier transform of a measurable function f with $\int_{-\infty}^{\infty} |f(x)| \, dx < \infty$. 5

Let $f \in L^1(\mathbf{R})$. Then prove that $\hat{f}$ is continuous on $\mathbf{R}$, where $\hat{f}$ is the Fourier transform of f.

(b) Define a component in a metric space. Show that every non-empty connected subset of a metric space is contained in a unique component. 5

6. (a) State Implicit Function Theorem and verify the theorem for the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by $f(x, y) = y^2 - yx^2 - 2x^5$ in the neighbourhood of the point $(1, -1)$. 4

(b) Find the Fourier Series for the function $f(t) = t^2, -\pi \leq t \leq \pi$. 4

(c) Show that the set $\left\{1, \frac{1}{2}, \frac{1}{3}......\right\}$ is no where dense in R with standard metric. 2

7. (a) Show that the system $R : S \to S$ given by 3

$$g(t) = (Rf)(t) = \int_{-\infty}^{t} f\,(\hat{j})\, d\,\tau,$$ is a time-invariant system.

(b) Obtain the second order Taylor's series expansion of the function f, defined by $f\,(x_1, x_2) = x_1^2 x_2 + 5x_1 e x_2$ at $(-1, 0)$. 5

(c) State Monotone Convergence theorem. Check whether the sequence $\{ f_n \}$ where $f_n = \chi_{[n,\ n+1]}$ $(n = 1, 2, ...)$ satisfies the conditions of Monotone Convergence theorem. 2