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MMT-002

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

June, 2011

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. 5 is* ***compulsory****. Answer* ***any three*** *questions from question Nos.* ***1*** *to* ***4****. Use of calculators is* ***not*** *allowed.*

1. (a) Let T be a linear operator from $\mathbf{R}^3$ to itself **3**

given by $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1+2x_2 \\ 2x_2 \\ -2x_1-2x_2-x_3 \end{bmatrix}$.

Find the matrix of T with respect to the

ordered basis $\left\{ \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

(b) Show that, if U is a Unitary matrix with integer entries then each row and each column of U will have exactly one non-zero entry which is 1 or -1. **2**

2. (a) Write the Jordan canonicla form for 2

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Find the least square solution of the smallest norm for the system $Ax = y$ where 3

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

3. (a) Why is the matrix $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$ not unitarily diagonalisable ? Find a unitary matrix U such that U*AU is upper triangular. 3

(b) Find let (e^A) if $A = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$. 2

4. Find the singular value decomposition of 5

$$\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$

5. Which of the following statements are true and which are false ? Give reasons for your answer.

2x5=10

(a) If u and v are eigen vectors of a matrix A, $u - v$ is also an eigen vector of A.

(b) Two similar matrices have the same minimal polynomial.

(c) All the entries of a positive semi-definite matrix are non-negative.

(d) If A is a Hermitian matrix, then singular values of A are its eigen values.

(e) If D is a diagonalisable $n \times n$ matrix and N is a nilpotent $n \times n$ matrix, then D and N commute.