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ET-101(A)

B.Tech. Civil (Construction Management)/ B.Tech. Civil (Water Resources Engineering) B.Tech (Aero space Engineering)

04534

Term-End Examination

June, 2011

ET-101(A) : MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory.*** *Use of calculator is permitted.*

1. Answer **any five** of the following : **5x4=20**

(a) Evaluate the following limits, if they exist :

(i) $\lim_{x \to 0} \frac{\tan x}{\sqrt{x+5} - \sqrt{5}}$

(ii) $\lim_{x \to 0} \frac{\ln(1+x) - x}{1 - \cos x}$

(b) Determine values of p and q for which the function f defined by :

$$f(x) = \begin{cases} px + q, & \text{when } x \leq 0 \\ 1 - \frac{2}{x^2+1}, & \text{when } x > 0 \end{cases}$$

is continuous at $x = 0$

(c) Find $\frac{dy}{dx}$ when

$x = e^t (\cos t + \sin t)$, $y = e^t (\cos t - \sin t)$

(d) If $y = e^{m\cos^{-1}x}$, prove that

$(1-x^2)\, y_2 - x\, y_1 - m^2 y = 0.$

Hence find y_{n+2}, using Leibnitz's theorem.

(e) Show that the function f defined

$f(x) = x^3 - 6x^2 + 15x - 10$

is strictly increasing in every interval.

(f) If $z^3 - x\, z - y = 0$, prove that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{x + 3\, z^2}{(x - 3\, z^2)^3}$$

2. Answer *any four* of the following : **4x4=16**

(a) Evaluate the following integrals :

(i) $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$

(ii) $\int e^x . \frac{2 + \sin 2x}{1 + \cos 2x}\, dx$.

(b) Evaluate :

(i) $\int_1^2 \frac{dx}{x(x^3+1)}$

(ii) $\int_0^2 [x+1]\, dx$, where $[t]$ denotes the greatest integer $\leq t$.

(c) Find the volume of the solid obtained by revolving the curve, $x = a\cos\theta$, $y = b\sin\theta$ about the axis of x.

(d) Taking 6 sub-divisions of the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, find the approximate value of $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x}\, dx$, using the Trapezoidal Rule.

(e) Solve the differential equation :

$(3x^2 + y^2)dy + (x^2 + 3y^2)dx = 0.$

3. Answer ***any four*** of the following : **4x4=16**

(a) If a, b, c are constants whose sum is 3 and the vector field

$\vec{F} = (\sin y + az)\,\hat{i} + (x\cos y - bz)\,\hat{j} + (cx - y)\,\hat{k}$ is irrotational, then find their values.

(b) Find div (grad ϕ), where $\phi = x^3 y^2 z^4$.

(c) For the function, $f = \frac{y}{x^2 + y^2}$,

find the value of the directional derivative making an angle 60° with the positive direction of the axis of x at the point $(-1, 1)$.

(d) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$ along the curve, $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

(e) If $\vec{F} = (2x^2 - 3z)\,\hat{i} - 2xy\,\hat{j} - 4x\,\hat{k}$, then evaluate $\iiint_V \nabla \times \vec{F}\, dV$, where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$.

4. Answer *any three* of the following : **3x6=18**

(a) Prove that

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$$

(b) Find the eigen values and the eigen vectors of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(c) Solve the following system of linear equations by the matrix method :

$x - y + 2z = 5$

$x + 2y + 3z = 14$

$2x - 3y + 2z = 2$

(d) Show that :

$V = \{(x, y, z) \mid x + y + z = 0\}$ is a subspace of R^3 and find a basis of V.