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BME-001

BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

June, 2011

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : ***All*** *questions are* ***compulsory****. Use of calculator is allowed.*

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1. Answer *any five* of the following : **5x4=20**

(a) Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-1}{x}$

(b) A function $f(x)$ is defined as follows :

$$f(x)=\begin{cases} x, & x<1 \\ 2-x & 1\leq x\leq 2 \\ -2+3x-x^2 & x>2 \end{cases}$$

Discuss its continuity and differentiability at $x=1$ and $x=2$.

(c) If $y=\tan^{-1}x$, then show that $(1+x^2)y_{n+1}+2nx\,y_n+n\,(n-1)y_{n-1}=0$

(d) If $x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$,

Show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$

(e) Show that the area between the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3} a^2$.

(f) Solve the differential equation (**any one**) :

(i) $(x^2 - y^2) \frac{dy}{dx} = xy$

(ii) $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

2. Answer *any four* of the following : **4x4=16**

(a) If $A = 2\hat{i} + 2\hat{j} - \hat{k}$, $B = 6\hat{i} - 3\hat{j} + 2\hat{k}$, find $A \times B$ and the unit vector perpendicular to both A and B. Also find the sine of the angle between A and B.

(b) A particle is acted on by constant forces $3\hat{i} + 2\hat{j} + 5\hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ and is displaced from a point whose position vector is $2\hat{i} - \hat{j} - 3\hat{k}$ to a point whose position vector is $4\hat{i} - 3\hat{j} + 7\hat{k}$. Calculate the work done.

(c) Show that the vector field

$$F = 2x\,(y^2 + z^3)\,\hat{i} + 2\,x^2\,y\,\hat{j} + 3x^2\,z^2\,\hat{k} \text{ is}$$

conservative. Find its scalar potential and the work done in moving a particle from $(-1, 2, 1)$ to $(2, 3, 4)$.

(d) Find the divergence of the vector field

$$V = (x^2\,y^2 + z^3)\,\hat{i} + 2xy\,\hat{j} + e^{xyz}\,\hat{k}.$$

(e) If a is a constant vector and

$$r = x\,\hat{i} + y\,\hat{j} + z\,\hat{k},$$

Show that curl $(a \times r) = 2\,a$

(f) Find curl F

when $F = 2xyz^3\,\hat{i} + x^2\,z^3\,\hat{j} + 3x^2\,yz^2\,\hat{k}$

3. Answer *any six* of the following : **6x3=18**

(a) Matrices A and B are such that

$$3A - 2B = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix} \text{ and}$$

$$-4A + B = \begin{bmatrix} -1 & 2 \\ -4 & 4 \end{bmatrix} \text{ Find A and B.}$$

(b) Express A as the sum of a symmetric and a skew symmetric matrix, where,

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$$

(c) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Show that $A^2 - 4A - 5I = 0$, where I and 0 are the unit matrix and the null matrix of order 3 respectively. Use this result to find A^{-1}.

(d) Verify that $\frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$

is an orthogonal matrix.

(e) Determine the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(f) Solve the following system of equations by Cramer's rule.

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$

(g) Determine the rank of the matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

(h) Find the eigen values of the following matrix.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

4. Answer *any four* of the following : **4x4=16**

(a) In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS

(ii) The student has opted neither NCC nor NSS.

(iii) The student has opted NSS but not NCC.

(b) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random.

(i) 1

(ii) 0

(iii) At most 2

bolts will be defective.

(c) In a certain factory producing cycle tyres, there is a small chance of 1 in 500 tyres to be defective. The tyres are supplied in lots of 10.

Using Poisson distribution calculate the approximate number of lots containing no defective, one defective, and two defective tyres, respectively, in a consignment of 10,000 lots.

(d) The diameter of an electric cable is assumed to be continuous random veriate with p.d.f (probability density function)

$$f(x) = 6x(1-x),\ 0 \leq x \leq 1$$

find the mean.

(e) Assuming that the diameters of 1000 brass plugs taken consecutively from a machine form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm, how many of the plugs are likely to be rejected if the approved diameter is $0.752 + 0.004$ cm ?

(f) A manufacturer intends that his electric bulbs have a life of 1000 hours. He tests a sample of 20 bulbs, drawn at random from a batch and discovers that the mean life of the sample bulb is 900 hours with a standard deviation of 22 hours. Does this signify that the batch is not up to the standard ?