00548

BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

June, 2011

CS-601 : DIFFERENTIAL AND INTEGRAL CALCULUS WITH APPLICATIONS

Time : 2 hours *Maximum Marks : 75*

Note : *Question no. 1 is **compulsory**. Attempt **any three** more questions from question No. **2 to 6**. Use of calculator is permitted.*

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1. (a) Select the correct answer from the four given alternatives for each part given below : **6**

(i) If $f(x) = \frac{x^2-1}{x+1}$, then domain of the function is :

(A) R, the set of real numbers.

(B) $[-1, \infty]$

(C) All real numbers except - 1

(D) none of these

(ii) If $y = x \tan^{-1} x$, then $\frac{dy}{dx} =$

(A) $x^2 \tan^{-1} x + \frac{x^2}{\sqrt{1+x^2}}$

(B) $\tan^{-1} x + \frac{x}{1+x^2}$

(C) $\tan^{-1} x - \frac{x}{1+x^2}$

(D) $(\tan^{-1} x)^2$.

(iii) $\frac{d}{dx}\left(\frac{e^x}{x}\right) =$

(A) $\frac{e^x}{x^2}$ (B) $\frac{e^x}{x^2} + \frac{1}{x} e^x$

(C) $\frac{1}{x} e^x - \frac{e^x}{x^2}$ (D) $e^x \log x$

(iv) $\int \operatorname{cosec}(3x+4) \cot(3x+4)\, dx =$

(A) $\frac{1}{2} \operatorname{cosec}(3x+4) \cot(3x+4) + C$

(B) $\frac{1}{3} \operatorname{cosec}(3x+4) + C$

(C) $-\frac{1}{3} \operatorname{cosec}(3x+4) + C$

(D) $\log \cos(3x+4)\ C$

(v) If $f(x) = (x-1)e^x + 1$ and $x \geq 0$ then $f(x)$ is :

(A) increasing function

(B) decreasing function

(C) Strictly increasing function

(D) Strictly decreasing function

(vi) $\lim_{x \to 0} \frac{\sin 4x}{x}$ is :

(A) 0 (B) ∞

(C) 1 (D) 4

(b) Fill in the blanks : 6

(i) $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx =$ __________.

(ii) $\lim_{x \to 0} \frac{x^2}{\sin x^2} =$ __________.

(iii) $\int 2 \sin \frac{x}{2} \cos \frac{x}{2} dx =$ __________.

(iv) The minimum value of the function $f(x) = (2x-1)^2 + 3$ is __________.

(v) If $y = \cos x^2$, then $\frac{dy}{dx} =$ __________.

(vi) The equation of the normal to the curve $y = x^3$ at (1, 1) is __________.

(c) For each of the following functions, find whether the function is monotonically increasing or monotonically decreasing or neither, on given interval. 2

(i) $f(x) = 2x^3 - 8$ on $[0, 3]$

(ii) $f(x) = 2 \sin x$ on $\left[0, \frac{\pi}{2}\right]$

(d) If $x = \tan(l_n\, y)$, prove that 4

$$(1+x^2)\, y_1 = y.$$

Using Leibnitz theorem, find y_{n+1}.

(e) The perimeter of a rectangle is 100 m. Find the length of its sides when the area is maximum. 3

(f) Find the value of 'b' for which the function 3

$$f(x) = \begin{cases} x^2+1 & \text{when } x < 2 \\ bx+\frac{2}{x} & \text{when } x \geqslant 2 \end{cases}$$

is continuous at $x = 2$.

(g) Evaluate ; 3

$$\int \left(\log x^3 + 9\sin^3 x\right) \left[27 \sin^2 x \cos x + \frac{3}{x}\right] dx$$

(h) Evaluate $\lim_{x \to \infty} \frac{11x^2 - 6x+8}{9x^2 - 5x+5}$ 3

2. (a) For what value of k is the function ; **3+4+4+4**

$$f(x) = \begin{cases} 2x+1, & x \le 2 \\ x+k, & x > 2 \end{cases}$$

is continuous at $x=2$

(b) Find $\frac{dy}{dx}$ if $y = \sin x \sin 2x \sin 3x$

(c) Evaluate ; $\int x^3 \log 2x \, dx$

(d) Prove that $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \frac{\pi}{4}$

3. (a) Find the n^{th} derivative of the following function $f(x) = (ax+b)^m$ where a and b are real numbers and m is a positive integer.

(b) Can Rolle's Theorem be applied to the following function ? **5+5+5**

$y = \sin^2 x$ on the interval $[0, \pi]$. Find 'C' such that $f'(C) = 0$, in case Rolle's theorem can be applied.

(c) Integrate any one of the following :

(i) $\int x^3 e^{2x} \, dx$

(ii) $\int \frac{1}{e^x+1} dx$

4. (a) Evaluate $\int_{0}^{4} e^{2x}\, dx$ **5+5+5**

(b) Compute the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

(c) If $z = e^{ax+by} f(ax - by)$, prove that

$$b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2\,abz$$

5. (a) Solve any one of the following : **5+5+5**

(i) $xy \frac{dy}{dx} = 1 + x + y + xy$

(ii) $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$

(b) For what value of 'k' is the following function continuous at $x = 1$?

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ k & x = 1 \end{cases}$$

(c) If $u = f(x - y, y - z, z - x)$, prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

6. (a) Calculate the radius and the height of a right circular cylinder of maximum volume which can be cut from a sphere of radius R.

(b) Solve any *one* of the following : **5+5+5**

(i) $\frac{dy}{dx} = e^{2x-3y} + 4x^2 e^{-3y}$

(ii) $(xy + x)\ dy - (xy + y)\ dx = 0$

(c) A river is 80 ft wide. The depth in feet at a distance x ft, from one bank is given by the following table

x :	0	10	20	30	40	50	60	70	80
d :	0	4	7	9	12	15	14	8	3

Using Simpson's $\frac{1}{3^{rd}}$ rule, find approximately the area of the cross-section.
