M.Sc. ACTUARIAL SCIENCE

00081

Term-End Examination

June, 2010

MIA-002 F2F : PROBABILITY AND MATHEMATICAL STATISTICS

Time : 3 hours *Maximum Marks : 100*

Note : *For* **Section – A**, *the student has to Attempt* **any five** *questions out of* **7** *questions and for* **Section – B**, *Attempt* **any four** *questions out of* **6** *questions.*

SECTION–A

(Attempt ***any five*** *questions and each question carry 8 marks)*

1. (a) The members of a consulting firm rent cars from three rental agencies. They have the following details : **5**

70% from agency 2 and 20% from agency 1 and 10% from agency 3. If 9% of the cars from agency 1 need a tune-up, 15% of the cars from agency 2 and 6% of the cars from agency 3 need a tune-up, and if a rental car is delivered to the consulting firm needs a tune-up, what is the probability that it came from agency 2 ?

(b) The probability that a student passes a mathematics test is $\frac{2}{3}$ and the probability that he passes both a mathematics test and a statistics test is $\frac{14}{45}$. The probability that he passes at least one test is $\frac{4}{5}$. What is the probability that he passes a statistics test ? 3

2. The medical department of ABC University, maintains records of yearly medical leave taken by its employees. The following are the number of days taken by 30 of its employees.

13, 47, 10, 3, 16, 7, 25, 8, 21, 19, 12, 45, 1, 8, 4, 6, 2, 14, 13, 7, 34, 13, 41, 28, 50, 14, 26, 10, 24. 36.

(a) Construct a stem-leaf display of the data. 3

(b) Construct frequency distribution by taking class intervals of the form 0–10, 10–20 and so on. 3

(c) Draw histogram and suggest whether the distribution is symmetrical, positively skewed or negatively skewed. 2

3. Let X be a continuous r.v with B.d.f given by

$$f_X(x) = \begin{cases} kx & ; 0 \le x < 1 \\ k & ; 1 \le x < 2 \\ -kx + 3k & ; 2 \le x < 3 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Determine the constant k. 3

(b) Find F (x). 4

(c) Find P $(x > 1.5)$. 1

4. In a large city A, 20% of a random sample of 900 school children had defective eye-sight. In an other large city B, 15% of random sample of 1600 children had the same defect.

(a) Is the difference between proportions significant ? 4

(b) Obtain 95% confidence limits for the difference in the population proportions. 4

5. (a) Find the moment - generating function of the discrete r. v x that has the probability distribution. 5

$$f(x) = \frac{1}{8}\binom{3}{x}, \; x = 0, 1, 2, 3$$

And Also find its mean and variance.

(b) If $X_1, X_2, ... X_n$ are random observations on a Bernoulli variate X, taking the value 1 with probability p and the value o with probability $(1-p)$, show that $\overline{X}(1-\overline{X})$ is a consistent estimator of $p(1-p)$. 3

6. Let X be a gamma random variable with parameters 2 and $\frac{1}{\theta}$, such that :

$$f(x) = \frac{1}{\theta^2} x e^{-x/\theta} \quad , x > 0$$

It is required to estimate θ based on $X_1, X_2, ... X_n$ a random sample of n observations of X, with mean $\overline{X}$.

(a) Write down the mean and variance of X. 2

(b) Show that the maximum likelihood estimator $\hat{\theta}$ is $\frac{1}{2}\overline{X}$ and find an expression for its Mean Square Error (MSE). 6

7. (a) The joint probability distribution of X and Y is given below : 5

y \ x	−1	+1
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient between X and Y.

(b) The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively. Find P $(X \geq 1)$. **2**

(c) Comment on "The mean of a binomial distribution is 3 and variance is 4. **1**

SECTION - B

(Attempt ***any 4*** questions and each question carries **15** marks)

8. Suppose that the random variable x follows on exponential distribution with probability density function.

$$f(x) = \lambda.\, e^{-\lambda x},\ 0<x<\infty\ (\lambda>0)$$

Let us define a new random variable $Y=X^{\frac{1}{3}}$.

(a) (i) Show that the cumulative distribution function of Y is given by **4**

$$F_y(y) = \begin{cases} 0 & ;\ y<0 \\ 1-\exp(-\lambda y^3), & y\geq 0 \end{cases}$$

Hence or otherwise find the probability density function of y.

(ii) Explain how would simulate a value of y given a value μ from the uniform U(0, 1) distribution. **3**

(b) (i) Find an expression for the maximum likelihood estimator of the parameter λ using a sample $y_1, y_2, ..., y_n$ from the distribution of y. **5**

(ii) Eight observed values of the random variable Y are given below : **3**

0.72, 1.15, 1.26, 1.03, 1.69, 1.30, 1.42, 1.15.

Calculate the maximum likelihood estimate of λ, using these values.

9. (a) Two researchers adopted different sampling techniques while investigating the same group of students to find the no. of students falling in different intelligence levels. The results are as follows : 10

No. of students in each level

Researchers	Below Average	Average	Above Average	Genius	Total
X	86	60	44	10	200
Y	40	33	25	2	100
Total	126	93	69	12	300

Test at 5% level, that whether the sampling techniques adopted by the two researchers are significantly different ?

(b) A co-efficient of correlation of 0.2 is derived from a random sample of 500 pairs of observations.

(i) Is this value of r significant ? 3

(ii) What are 95% and 99% confidence limits to the correlation coefficient in the population ? 2

10. A rubber manufacturing company has three plants at A, B and C. To measure how many employees at these plants know about total quality management, a random sample of six employees were selected and a quality awareness test was administered. The test scores are given below :

Test Scores

Observation no.	Plant A	Plant B	Plant C
1	85	71	59
2	75	75	64
3	82	73	62
4	76	74	69
5	71	69	75
6	85	82	67

(a) Write down the assumptions for analysis of variance. 3

(b) Test at 5% level, the null hypothesis that the average test scores are the same for all the three plant. 8

(c) Obtain 95% confidence interval of the population mean for the plant at A, assume that the variances of the score in the plants are same. 4

11. The following data gives details about number of hours 10 students studied for a statistics test and their scores in the test.

S. No.	Hours studied (x)	Test Score (y)
1	4	31
2	9	58
3	10	65
4	14	73
5	4	37
6	7	44
7	12	60
8	22	91
9	1	21
10	17	84

A regression model $Y = \alpha + \beta X + \epsilon$;

$$\epsilon \sim N(\mu, \sigma^2)$$

is to be fitted on the above data.

(a) Display the data in a scatter diagram and comment on the selection of a linear model for regression. **3**

(b) Fit the regression line of the test scores on the no. of hours studied. **5**

(c) Predict the average test scores of a person who studied 14 hrs for the test. **1**

(d) Test the hypothesis : $H_0 = \beta = 3$ against $H_1 : \beta > 3$ at the 0.01 level of significance. **4**

(e) Construct a 95% confidence interval for β. **2**

12. (a) In a certain experiment to compare two types of animal foods A and B, the following results of increase in weights were observed in animals :

S.No.	Increase in weight (in *lb*)	
	Food A (X)	Food B (Y)
1	49	52
2	53	55
3	51	52
4	52	53
5	47	50
6	50	54
7	52	54
8	53	53
Total	407	423

(i) Assuming that two samples of animals are independent, test at 5% level of significance, that whether Food B is better than food A ? 5

(ii) Also test at 5% level, when the same set of eight animals were used in both the foods, whether Food B is better than Food A ? 6

(b) A soft drink vending machine is set that the amount of drink dispensed is a r.v with mean of 200 millilitres and standard deviation of 15 millilitres. What is the probability that the average (mean) amount dispensed in a random sample of size 26 is at least 204 millilitiers ? 4

13. (a) Two random samples gave the following results : 8

Sample No.	Size	Sample mean
1	10	15
2	12	14

and given $\sum_{i=1}^{10}(X_1-\overline{X}_1)^2 = 90$

$\sum_{i=1}^{12}(X_2-X_2)^2 = 108$

Test for the equality of means at 5% level of significance assuming normality of population ?

(b) Define :

(i) Type I Error. **1**

(ii) Type II Error. **1**

(iii) The size of a test. **1**

(iv) The power of a test. **1**

(c) Let $x_1, x_2, \ldots x_n$ be a random sample from normal population N $(\mu, 1)$ show that $\frac{1}{n} \Sigma x i^2$ is an unbiased estimator of $(\mu^2 + 1)$. **3**
