

M.Sc. ACTUARIAL SCIENCE

Term-End Examination

June, 2010

00561

MIA-001 F2F : FINANCIAL MATHEMATICS

Time : 3 hours

Maximum Marks : 100

Note : In addition to this paper you should have available Actuarial Tables and an electronic calculator.

SECTION - A

Answer *any five* questions :

5x8=40

1. Given $d=7\%$, find :

(a) $\ddot{G}_{\overline{5}|}$ 2

(b) $(Da)_{\overline{10}|}$ 2

(c) $S_{\overline{6}|}^{(3)}$ 2

(d) $(\bar{Ia})_{\overline{7}|}$ 2

2. (a) Define real rate of interest. 1

(b) Given positive inflation, state whether real rate of interest will be greater than money rate of interest. 1

- (c) A borrower is under an obligation to repay a bank Rs. 6280 in four years time, Rs. 8460 in seven years time and Rs. 7350 in thirteen years time. 6

As part of a review of his future commitment the borrower now offers either :

- (i) To discharge his liability for these three debts by making an appropriate single payment five years from now, or
(ii) To repay the total amount owed (i.e.. Rs. 22090) in a single payment at an appropriate future time.

On the basis of an effective rate of 8% per annum find the appropriate single payment if offer (a) is accepted by the bank, and the appropriate time to repay the entire indebtedness if offer (b) is accepted.

3. (a) Prove that $\bar{\delta}_{\overline{m}|} = \frac{(1+i)^n - 1}{\delta}$. 3

- (b) Ramesh is looking to invest in one of the following projects : 5

Cash flow for project A are :

Initial cost - Rs. 100000 as at 1st Jan 2010

00561

Annual income - Rs. 7500 at the end
of 1st year,
increasing at 5% per
annum till 31 Dec
2020.

Residual value - Rs. 150000 as at 31
Dec 2020.

Cash flow for project B are :

Initial cost - Rs. 100000 as at 1st
Jan 2010

Annual income - Nil

Residual value - Rs. 300000 as at 31
Dec 2020.

Calculate the NPV for each project as at 1st
Jan 2010 at an interest rate of 10% per
annum and advice Ramesh in choosing any
one project based on NPV.

4. (a) An investor is to make payment of Rs. 100 4
annually in arrears for six years followed
by Rs. 200 per annum payable half yearly
in arrears for a further six years. The
investment earn interest at 8% per annum
convertible half yearly. Calculate the
accumulated amount at the end of 14 years.

- (b) A credit company offers a loan of Rs. 5000. The loan is to be repaid over 4 year term by level monthly instalments of Rs. 130, payable in arrears. What is the APR for the transaction ? 4
5. (a) Define the following terms :
- (i) arbitrage 3
 - (ii) hedging 1
- (b) A 9-month forward contract is issued on 1st April 2004 on a stock with a price of Rs. 7 per share. Dividend of 50 p per share are expected after 2 and 8 month. 4
- Assuming a risk free effective rate of interest of 6% per annum and no arbitrage calculate the forward price.
6. The following n-year spot rates were observed at time $t=0$.
- 1 year spot rate of interest 4%
 - 2 year spot rate of interest 5%
 - 3 year spot rate of interest 6%
 - 4 year spot rate of interest 7%
 - 5 year spot rate of interest 7.5%
 - 6 year spot rate of interest 8%

- (a) Define what is meant by an n-year spot rate of interest ? 2
- (b) Calculate the two year forward rate of interest at time $t=3$. 2
- (c) Using the above n-year spot rates calculate the 6-year par yield at time $t=0$. 4

7. A man invested Rs. 20000 at time 1, 3 and 4. In return he received Rs. 8000 per annum monthly in arrear for 15 years starting at time 4. Calculate the discounted payback period and the accumulated profit after he has received his last monthly payment, assuming an effective rate of interest of 6%. 8

SECTION - B

Answer *any four* questions :

4x15=60

8. The force of interest at time t is given by :

$$\delta(t) = \begin{cases} 0.01 + 0.01t & 0 \leq t < 4 \\ 0.15 - 0.003t^2 & 4 \leq t < 6 \\ 0.06 & t \geq 6 \end{cases}$$

- (a) Find expression for the value at time $t=0$ of a payment of Rs. 100 at time t . 5
- (b) A payment of Rs. 100 is received at time 3 and a further payment of Rs. 250 is received at time 5. Calculate the accumulated value of these payments at time 8. 5
- (c) The rate of payment at time t is $\rho(t) = 10 + 10t$ for $3 < t < 4$. Find the total accumulated value of these payments streams by time 10. 5
9. (a) An annuity is payable for 20 years. The amount of the annuity in the t^{th} year is Rs. t^2 . On the basis of an effective rate of interest of 5% per annum find the present value, assuming that it is payable annually in advance. 5

- (b) A loan of Rs. 2000 is repayable by a level annuity certain payable annually in arrear for eighteen years. The amount of the annual repayment is calculated on the basis of an annual interest rate of 10% for the first six years and 9% there after. Find :
- (i) The amount of the annual repayment. 3
 - (ii) The amount of capital contain in the fourth repayment and the twelfth repayment. 5
 - (iii) Immediately after making the twelfth repayment the borrower makes an additional capital repayment of Rs. 100, the amount of the annual repayment being appropriately reduced. Assuming that the interest basis is unaltered find the amount of the revised repayment. 2

10. (a) Explain briefly the followings : 10
- (i) Debenture
 - (ii) Ordinary shares
 - (iii) Convertibles
 - (iv) Margin
 - (v) Interest rate swaps

- (b) A loan of Rs. 25000 was issued and was repayable at par after three years. Interest was paid on the loan at the rate of 8% per annum, payable annually in arrear. The value of the retail price index at various time was as follows :

At the date the loan was made 205.0

One year later 215.6

Two years later 223.5

Three years later 231.5

Calculate the real rate of return on the loan.

11. A loan of nominal amount of Rs. 100000 is to be issued bearing interest payable quarterly in arrear at the rate of 8% per annum. Capital is to be redeemed at 105% on a coupon date between 15 and 20 years after the date of issue, inclusive, the date of redemption being at the option of the borrower.

- (a) An investor who is liable to income tax at 40% and tax on capital gain at 30% wishes to purchase the entire loan at the date of issue. What price should she pay to ensure a net effective yield of at least 6% per annum ?

- (b) Exactly 10 months after issue, the loan stock is sold to an investor who pay income tax at 20% and capital gain tax at 30%. Calculate the price this investor should pay to achieve a yield of 6% per annum on the loan : 6
- (i) assuming redemption at the earliest possible date.
- (ii) assuming redemption at the latest possible date.
- (c) Explain which price the investor should pay to achieve a yield of at least 6% per annum. 2
12. In any year, the yield on funds invested with a given insurance company has a mean j and standard deviation s , and is independent of the yields on all previous years. Let i_t be the rate of interest earned in the t^{th} year.
- (a) Derive formula for the mean and variance of the accumulated value after n -years of a single investment of 1 at time 0. 6
- (b) Each year the value of $(1 + i_t)$ is log-normally distributed. The mean and standard deviation of i_t are 0.07 and 0.20 respectively.
- (i) Determine the parameter μ and σ^2 of the log-normal distribution of $1 + i_t$. 5

- (ii) Determine the distribution of S_{15} , where S_{15} denotes the accumulation of one unit of money over 15 years. 2
- (iii) Determine the probability that $S_{15} > 2.5$. 2

13. (a) Let $\{C_{t_k}\}$ denotes a series of cash flow payments at time t_k for $k=1, 2, \dots, n$, and $P(i)$ denotes the present value of these payments at an effective interest rate i , so that :

$$P(i) = \sum_{k=1}^n C_{t_k} / (1+i)^{t_k}$$

- (i) Define the discounted mean term of the cash flow in terms of t_k , C_{t_k} and V . 1
- (ii) Define the volatility of cash flows in terms of $P(i)$ and show that for this series of cash flows the discount mean term = volatility $\times (1+i)$. 3
- (b) A fund has to provide an annuity of Rs. 60000 per annum payable yearly in arrear for the next 9 years followed by a 11

final payment of Rs. 750000 in 10 years time.

The fund has earmarked cash assets exactly equal to the present value of the payments and the fund manager want to invest in two zero coupon bonds : Bond A which is repayable at the end of 5 years and Bond B which is repayable at the end of 20 years.

The fund manager wants both the assets and the liabilities to have the same volatility. To achieve this, how much should the manager invest in each of the bonds, given that an effective rate of interest of 7% per annum is used to value both assets and liabilities ?
