No. of Printed Pages : 5 **MMTE-001**

00690

M.Sc. (Mathematics with Applications in Computer Science) (MACS)

Term-End Examination

June, 2010

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. **1** is **compulsory**. Do **any four** questions out of question No. **2** to **7**. Calculators are not allowed.*

1. Prove on disprove the following statements : **5x2=10**

   (a) If every vertex of a simple graph G has degree 2, then G is a cycle.

   (b) Every bipartite graph need not be a tree.

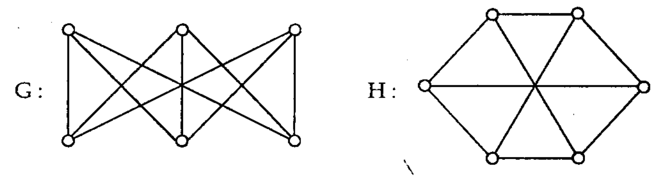
   (c) The complete bipartite graph $K_{3,4}$ is Eulerian.

   (d) Every edge cut is a disconnecting set.

   (e) Any simple graph with at least 4 vertices is 4 - colourable.

2. (a) If u and v are the only vertices of odd degree in a graph G, prove that G contains a u-v path. **2**

(b) Define isomorphism between graphs and check whether the following two graphs are isomorphic : **4**

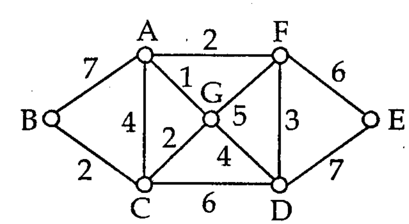


(c) State a necessary and sufficient condition for a graph to be bipartite. Prove the sufficiency of the condition. **4**

3. (a) Check whether the sequence (4, 4, 4, 2, 2, 2) is a graphic sequence ? If yes provide a construction. **2+1=3**

(b) If G is an n-vertex connected graph that has no cycles, prove that G has n-1 edges. **3**

(c) Using Dijkstra's algorithm, find the shortest distance from vertex A to all the vertices in the following weighted graph. **4**

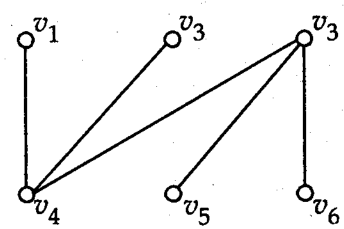


4. (a) In the graph given below give the following with justification : **3**

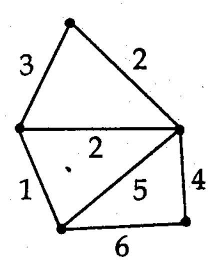
(i) A matching of maximum size

(ii) A vertex cover of minimum size

(iii) An independent set of vertices of maximum size.

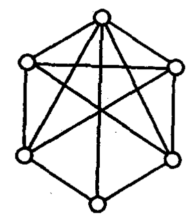


(b) Find the minimum spanning tree in the following connected weighted graph. **3**



(c) If G is a simple graph, prove that $k(G) \leq k'(G)$ where $k(G)$ is vertex connectivity of G, $k'(G)$ is edge connectivity of G. **4**

5. (a) Find the chromatic number X(h) to the following graph. 3



(b) Show that for any graph G with n vertices the chromatic number $X(G) \geq \frac{n(G)}{\alpha(G)}$ when n(G) the clique number and $\alpha(G)$, the independence number. 3

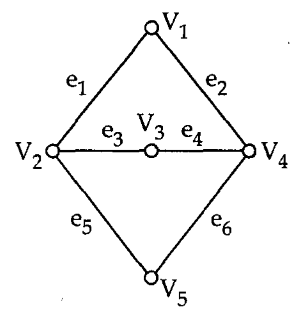
(c) State and prove Euler's formula for a planar graph. 4

6. (a) Show that the graph formed by deleting one edge from K_{33} is planar 3

(b) Use complete graphs and counting arguments to prove that : 3

$$\binom{n}{2} + \binom{k}{2} + k(n-k) + \binom{n-k}{2} \text{ for } 0 \leq k \leq n.$$

(c) Find the adjacency and incidence matrices of the following graph. 4



7. (a) State Dirac's Theorem for Halmiltonian graph. Is the converse true ? Justify your answer. 4

(b) Prove that K_{33} is non-planar. 3

(c) Show that in a graph $G, S \leq V(G)$ is an independent set if and only if $V(G) \backslash S$ is a vertex cover of G. 3