No. of Printed Pages : 8 **MMT-008**

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

00311

Term-End Examination

June, 2010

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 8 is* ***compulsory****. Answer* ***any six*** *questions from question number* **1** *to 7. Use of calculator is* ***not*** *allowed.*

1. (a) If $\{x\,(+) : t > 0\}$ is a Poisson process with rate λ and Sm denotes the duration from start to the occurrence of m^{th} event, obtain the distribution of Sm. If $\lambda = 1$ per hour then find probability that duration from start to the occurrence of third event will be less than 2 hours. **8**

(b) Suppose that random variables X and Y have the following joint p.d.f. **7**

$$f(x, y) = \begin{cases} x+y \ , & 0<x<1, 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find the conditional p.d.f. of X given $Y = y$.

(ii) Check independence of X and Y.

(iii) Compute $P\left[0 < X < \frac{1}{3} \middle| Y = \frac{1}{2}\right]$.

2. (a) Customers arrive at a counter in a bank in accordance to Poisson process at an average rate of 8 per hour. Service time at the counter follows exponential distribution with a mean 5 minutes. Find : 8

(i) the probability that a customer will have to wait before service.

(ii) proportion of time the counter will be idle.

(iii) the probability that total time spent at the counter by a customer is more than 10 minutes.

(iv) average waiting time at the counter.

(b) In a Branching process, the offspring distribution is given as : 7

$$p_k = \binom{n}{k} p^k q^{n-k}, \quad k = 0, 1, 2, \ldots n; \; q = 1 - p$$
$$0 < p < 1$$

Find the probability of ultimate extinction of the process given that (i) $n = 2$, $p = .2$ (ii) $n = 2$, $p = .8$,

3. (a) Of the three bags containing red and green balls, the first bag has 6 red and 4 green balls, the second bag has 3 red and 7 green balls and the third bag has 2 red and 8 green balls. A bag is selected at random and from the selected bag, a ball is chosen at random and is found to be green. Find the probability that the ball came from the second bag. 6

(b) Customers in a bank may get service from any one of the two counters. Customers arrive in the bank according to Poisson law at the rate 20 per hours. Service time at each counter is supposed to follow exponential distribution with mean 4 minutes. As an alternative bank thinks to install an automatic servicing machine which although has single service channel, but it will be able to serve two times faster than the counter clerk. Which system will be better in terms of customer waiting time in the bank. 9

4. (a) The probability of a dry day following a rainy day is .25 and probability of a rainy day following a dry day .20 in a two state simple weather model. Write down the transition probability matrix p for this two-state Markov chain P and find $p^{(3)}$, the three step transition probability matrix. If the probability of rainy day on August 1 be .7 then find the probability that August 4 will be (i) rainy day (ii) dry day. 8

(b) Let $\{X_n, n=1, 2, ...\}$ be i.i.d. geometric random variables with probability mass function $p(X_n = i) = (1-p)p^{i-1}, i = 1, 2, 3...$ 7

Find the renewal function of the corresponding renewal process.

5. (a) Let $\overline{X} = \{X_1, X_2, X_3\}$ be a random vector and X be the data matrix given below 8

$$X' = \begin{bmatrix} 5 & 2 & 5 \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{bmatrix} : \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Find (i) Variance-covariance matrix Σ.

(ii) Correlation matrix R.

(b) Let X denote the data matrix for a random sample of size 3 from a bivariate normal population, where 7

$$X = \begin{bmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{bmatrix}$$

Test the hypothesis Ho : $\underset{\sim}{\mu} = (9, 5)'$ at 5% level of significance.

6. (a) On the basis of past experience about the sales (X1) and profits (X2) the population mean vector and variance-covariance matrix for the industry was as given below 7

$$\mu = \begin{bmatrix} 30 \\ 10 \end{bmatrix} \quad \Sigma = \begin{pmatrix} 10 & 5 \\ 5 & 4 \end{pmatrix}$$

From a sample of 10 industries the sample mean vector was found as below

$$\overline{X} = \begin{bmatrix} 33 \\ 7 \end{bmatrix}$$

Test whether the sample confirms the truthfulness of the industry claim of population mean vector.

(you may like to use the following values :

$\chi^2_2, .05 = 5.99, \chi^2_3, 0.05 = 7.81$).

(b) Suppose $\underset{\sim}{x} = (x_1, x_2, x_3)'$ be distributed as a trivariate normal distribution, **8**

$$N_3(\underset{\sim}{\mu}, \Sigma), \text{ where } \underset{\sim}{\mu} = (2, 1, 3)', \Sigma = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}.$$

Find the distribution of $\underset{\sim}{u} = \begin{bmatrix} X_1 - X_3 \\ X_1 + X_3 - 2X_2 \end{bmatrix}$.

7. (a) Let X and Y be a pair of independent random variables. Find the Laplace transform of $Y = X_1 + X_2$. **7**

(b) On the basis of 7 variables $X_1, X_2, \ldots, X_7$ a factor analysis was performed. The factor loadings of first two factors obtained through principal components method with varimix rotation alongwith communality and variance summerized are given below **8**

Variables	Factors		Communality
	I	II	
X_1	.697	.476	.712
X_2	.748	.445	.758
X_3	.831	.350	.813
X_4	.596	.648	.775
X_5	.935	.160	.899
X_6	.866	.201	.790
X_7	.166	.936	.903
Sum of squares	3.473	1.909	
Variance summerized	.535	.277	Average .807

(i) Write linear equations for all the two factors.

(ii) Explain how the values of communality, variance summerized.

(iii) Interpret the loading coefficients and variance summerized.

8. State whether following statements are *true* or *false*. Justify your answer : **2x5=10**

(a) Posterior probabilities obtained from Bayes theorem are larger than respective prior probabilities.

(b) The probability of extinction of a branching process is the root of $S = P(S)$, where $P(S)$ is the p.g.f. of the offspring distribution.

(c) Laplace transform of a function completely characterises the function.

(d) A state in a Markov chain is persistent, if the return to that state is not certain.

(e) Given A a nonsingular matrix, T^2 statistics will be invariant to the transformation of linear form.
