# Master's in Mathematics with Applications in Computer Science M.Sc. (MACS)

00447

## Term-End Examination June, 2010

## MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

*Time : 2 hours* *Maximum Marks : 50*

***Note :*** *Question No.* **1** *is* ***compulsory.*** *Do any* ***four*** *questions out of question nos.* **2** *to* **7**. *All computations may be kept to* **3** *decimal places. Use of calculators is* ***not*** *allowed.*

1. State whether the following statements are ***true*** or ***false***. Justify your answer with the help of a short proof or a counter example : **2x5=10**

   (a) For the differential equation $x^3y'' + xy' + 2y = 0$, $x = 0$ is a regular singular point.

   (b) If g $(x)$ is a polynomial of degree $k < n$, then

   $$\int_{-1}^{1} g(x)\, P_n(x)\, dx = \frac{2}{2n+1},$$

   where $P_n(x)$ is the Legendre polynomial.

(c) The inverse Fourier transform

$$f^{-1}\left[\frac{1}{\alpha^2+2\alpha+5}\right]=\frac{1}{4}e^{-[2|x|+i\,x]}.$$

(d) The interval of absolute stability of the Runge-Kutta method

$$Y_{i+1}=y_i+\frac{1}{2}(k_1+k_2),$$

$k_1=h\,f(x_i, y_i)$, $k_2=h\,f(x_i+h, y_i+k_1)$ is $-2<\lambda h<0$.

(e) The function $y(t)$ satisfying the integral equation $y(t)+\int_0^t y(z)(t-z)\,dz=t$, is $y(t)=\sin t$.

2. (a) Using Laplace transform method solve the following simultaneous equations 6

$$\frac{dx}{dt}+\frac{dy}{dt}=t$$

$$\frac{d^2x}{dt^2}-y=e^{-t}$$

Subject to conditions $x(0)=3$, $x^1(0)=-2$, $y(0)=0$.

(b) Derive the Fourier-Bessel series for $f(x)=x$, $0\le x\le 1$, in terms of the functions $J_1(\lambda n\,x)$, where $\lambda_n$ are the zeros of $J_1(x)$. 4

**3.** (a) Find the power series solution, near $x=0$, of the differential equation. **6**

$$9x\,(1-x)\,y''-12\,y'+4y=0.$$

(b) Derive the constants in the method **4**

$y_{i+1}=a\,y_{i+2}+h(b_1 y'_i+b_2 y'_{i-1}+b_3 y'_{i-2}+b_4 y'_{i-3})$ for the solution of $y'=f(x,y)$. Determine also the truncation error and the order of the method.

**4.** (a) Solve the initial value problem **7**

$$y'=x^2+\sqrt{y}+1,\; y\,(0)=1$$

upto $x=0.6$, using the predictor-corrector method

$$\text{P}: \quad y_{n+1}^{(p)}=y_n+\frac{h}{2}\,(f_n-f_{n-1})$$

$$\text{C}: \quad y_{n+1}^{(c)}=y_n+\frac{h}{12}\,[\,5f\,(x_{n+1},\,y_{n+1}^{(p)}\,+8\,f_n-f_{n-1})$$

with step length $h=0.2$ compute the starting value using Euler's method and perform two corrector iterations per step.

(b) Evaluate $z\,[t^2\,u\,(t-3)]$, where $u$ represents the unit step function. **3**

5. (a) Find the solution of the initial, boundary value problem 5

$u_t = u_{xx}$, $0 \le x \le 1$,

$u(x, o) = \sin(2\pi x)$, $0 \le x \le 1$,

$u(0, t) = 0 = u(1, t)$,

Using the Crank-Nicolson method with $\lambda = 0.6$. Assume $h = 1/3$. Integrate for one time level.

(b) Construct Green's function for the following boundary - value problem. 5

$$\frac{d^2 y}{dx^2} + 9y = 0,\ y(0) = y(1) = 0.$$

6. (a) Using generating function for Legendre polynomial $P_n(x)$, show that 4

$$\frac{P_{n+1}(x) - P_{n-1}(x)}{2n+1} = C + \int P_n(x)\, dx$$

Where C is a constant.

(b) Using Galerkin method with triangular elements and one internal node, find the solution of boundary value problem $\nabla^2 u = 0$ in R, 6

Where $u = x + y$, on the boundary. Take $h = \frac{1}{2}$ and R is the square $0 \le x \le 1$, $0 \le y \le 1$.

7. (a) Solve the boundary value problem 7

$$y'' - 5y' + 4\,y = 0$$

$y(0) - y'(0) = -1, \quad y\,(1) + y'(1) = 1$

Using second order finite differences for $y'$ and $y''$, with $h = 1/2$.

(b) Evaluate $\int J_1\,(x) \sin x \, dx$. 3