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MMT-005

MASTER'S IN MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE M.Sc. (MACS)

00278

Term-End Examination

June, 2010

MMT-005 : COMPLEX ANALYSIS

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. 1 is **compulsory**. Attempt **any three** questions from question number 2 to 5. Use of calculator is **not** allowed.*

1. State with reasons whether the following statements are ***true*** or ***false*** : **2x5=10**

(a) Geometrically the set of all complex numbers z such that $z + \bar{z} = |z|^2$ represents a circle.

(b) Let $s = \{z : |z| < 1 \text{ or } |z-1| < 1\}$ then s is a domain.

(c) $f(z) = \sin z$ is bounded in the complex plane.

(d) The zeros of $\cos\left(\frac{1}{z}\right)$ are isolated.

(e) The radius of convergence of the series $\sum_{n=1}^{\infty} 2^n z^{n!}$ is 1.

2. (a) Show that the following function is not continuous at $z=0$. 2

$$f(z) = \begin{cases} \dfrac{\text{Re}(z)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

(b) Show that $u(x, y) = x^2 - y^2$ is harmonic in the whole plane. Find its harmonic conjugate and hence the associated analytic function. 3

3. (a) Evaluate $\int_C \frac{4}{(4z^2 - 1)} dz$; 2

where C is the unit circle with Centre 1.

(b) Prove that the transformation $w = \frac{1}{z}$ transforms circles and lines into circles and lines. In particular, find the transformation of the line $y=1$ under the above transformation. 3

4. (a) Discuss the nature of singularities of $f(z) = \frac{1}{z^3 \sin z}$. 2

(b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$. 3

5. (a) Prove that $Q(z) = \frac{2z-1}{2-z}$ is a conformal map which maps the closed unit disk $\overline{D} = \{z : |z| \leq 1\}$ onto itself. 3

(b) Evaluate $\int_C \frac{e^z}{(z-i)^3} dz$, where C is the eight-like figure given below : 2