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MMT-003

00135

MASTER'S IN MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE M.Sc. (MACS)

Term-End Examination

June, 2010

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is* ***compulsory****. Answer* ***any four*** *questions from Q. No.* ***2*** *to Q. No.* ***6****. Use of calculators is* ***not*** *allowed.*

1. Which of the following statements are ***true*** and which are ***false*** ? Give reasons for your answer : **10**

(a) The dihedral group D_4 has five elements of order two.

(b) In a group of odd order, the number of conjugacy classes is also odd.

(c) A free group on two generators has sub groups of order n. for every positive integer n.

(d) Z_4 is the unique field with 4 elements.

(e) A finite group of order 12 can have a irreducible representation of degree 3 or degree 4, but not both.

2. (a) Show that, upto isomorphism, there is a unique group of order 33. 4

(b) Let H be the group of quarternions $\{\pm 1, \pm i, \pm j, \pm k\}$ show that, $P : H \to GL_2(C)$ defined by 4

$$P(i) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, P(j) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, P(k) = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

is a group representation. Find the character of p and hence check that it is an irreducible representation.

(c) For a finite field F of 11 elements show that $F\left[\sqrt{5}\right]$ and $F\left[\sqrt{7}\right]$ are not the same. 2

3. (a) Let α, β be complex numbers. Prove that, if $\alpha + \beta$ and $\alpha\,\beta$ are algebraic numbers, then α and β are also algebraic. 4

(b) For a matrix $A \epsilon SU_2$, if the entry a_{11} is a complex number of modulus 1, then show that $a_{12} = 0$. 3

(c) Find two elements of order 2 in the dihedral group D_4 such that they generate D_4. 3

4. (a) Find all the irreducible representations of S_3. 4

(b) Show that the sylow 2-subgroup of the dihedral group D_6 is the Klein group. 6

5. (a) Let G be a free group on n generators, n>1. Show that there are infinitely many subgroups of G which are again free on n generations. **4**

(b) Show that the polynomial $x^3 + 2x + 1$ over the finite field $\mathbf{F}_3$ is irreducible. **2**

(c) Show that a finite group with exactly 2 conjugacy classes is of order 2. **4**

6. (a) Let ρ he a representation of a finite group G on a complex vector space V. Show that there exists a positive definite, hermitian form $< , >$ on V such that $<v, w> = <e_g(v), e_g(w)>$ for $v, w \in V$ and $g \in h$ where $\rho_g (v) = (\rho (g))(v)$. **6**

(b) If $\rho \in SO_3(\mathbf{R})$, show that 1 is an eigen value of ρ. **4**

7. (a) What are the possible degrees of irreducible polynomials that divide $x^{64} - x \in \mathbf{F}_2 [x]$? Justify your answer. **3**

(b) Let $N = \{1, (1\ 2\ 3), (1\ 3\ 2)\}$ be the normal subgroup of S_3. Find the character of the representation ρ obtained by the action of S_3 on N by conjugation. **3**

(c) Prove that two elements a, b of a group generate the same subgroup as bab^2 and bab^3. **2**

(d) Show that $i \notin Q(\sqrt{2})$. **2**
