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00288

MASTER'S IN MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE M.Sc. (MACS)

Term-End Examination

June, 2010

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. 5 is* ***compulsory****. Answer* ***any three*** *questions from question Nos. 1 to 4. Use of calculators is* ***not*** *allowed.*

1. (a) Let B and B′ be ordered bases of the vector space $\mathbf{R}^2$, where **2**

$$B = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \end{bmatrix} \right\} \text{ and } B' = \left\{ \begin{bmatrix} -7 \\ 9 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \end{bmatrix} \right\}.$$

Find the change of coordinates matrix from B′ to B.

(b) Show that the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is positive semidefinite. Also find its square root. **3**

2. (a) Prove that the geometric multiplicity of an eigenvalue cannot exceed its algebraic multiplicity. 2

(b) The owner of a rapidly expanding business finds that for the first four months of the year her sales are (in lakhs) Rs. 3.0, Rs. 6.3, Rs. 11.3 and Rs. 18.5, respectively. She plots these figures on a graph and conjectures that for the rest of the year her sales curve can be approximated by a quadratic polynomial. Find the least squares quadratic polynomial to fit the data. 3

3. (a) Consider the matrix $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$. 3

Find the Jordan canonical form of A. Also find the matrix P such that $P^{-1}AP$ is in Jordan form.

(b) Find the QR-decomposition of the matrix 2

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

4. Construct a singular value decomposition of the matrix **5**

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}.$$

5. Which of the following statements are *true* and which are *false* ? Give reasons for your answers. **10**

(a) Eigenvectors corresponding to different eigenvalues are linearly independent.

(b) For a matrix A, rank (A) = rank (A^t A).

(c) The generalized inverse of a matrix is unique.

(d) The matrix $\begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalisable.

(e) If λ_1, λ_2 are the eigenvalues of the matrix

$\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$, then $|\lambda_1|^2 + |\lambda_2|^2 = 6$.