# Diploma in Civil Engineering / Diploma in Electrical and Mechanical Engineering

03715

## Term-End Examination

## June, 2010

## BET-021 : MATHEMATICS-II

*Time : 2 hours* *Maximum Marks : 70*

***Note :*** *Question number **1** is **compulsory**. Attempt **any four** questions out of the remaining questions. Use of calculator is permitted.*

1. (A) Select the correct answer from the **four** given alternatives : **7x1=7**

(a) $\int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right) dx$ is :

(i) 1

(ii) $x+c$

(iii) $\log\,(1+\cos 2x)$

(iv) $\log\,(1+\cos 2x)+c$

(b) $\frac{d}{dx}\left[\cos\left(\log x\right)\right]$ is

(i) $\sin (\log x)$

(ii) $-\sin (\log x)$

(iii) $-\sin (\log x). \frac{1}{x}$

(iv) $\sin (\log x). \frac{1}{x}$

(c) if $f(x) = \cos x$ and $0 < x < \pi$ then $f(x)$ is

(i) decreasing function

(ii) increasing function

(iii) strictly decreasing function

(iv) none of these

(d) if $Z_1$ and $Z_2$ are two complex numbers, then $|Z_1 + Z_2|$ is :

(i) $\geqslant |Z_1| - |Z_2|$

(ii) $\leq |Z_1| + |Z_2|$

(iii) $\geqslant |Z_1| + |Z_2|$

(iv) none of these

(e) If X and Y are two matrices so that

$$X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}, \; Y = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \text{ then } X =$$

(i) $\begin{pmatrix} 4 & 2 \\ -2 & 6 \end{pmatrix}$

(ii) $\begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 & 2 \\ -6 & 2 \end{pmatrix}$

(iv) none of these

(f) If $f(x) = \dfrac{x+1}{2x-1}$, $x \in R$ then Range of $f(x)$ is :

(i) R

(ii) R except $\dfrac{1}{2}$

(iii) R except 1

(iv) none of these

(g) $f(x)$ is a function and for $x=a$, $f'(a)=0$, $f''(a)=5$, then $f(x)$ for $x=a$ has a :

(i) local maxima

(ii) local minima

(iii) none of these

(B) Fill in the blanks : **7x1=7**

(a) $\int_0^1 \frac{\tan^{-1}}{1+x^2} dx$ = ________.

(b) $\frac{d}{dx}$ $[\tan^{-1}(\cot x) + \cot^{-1}(\tan x)]$ = ________.

(c) Every entry in the principal diagonal of a skew symmetric matrix is ________.

(d) Three points $(x_1,y_1)$ ; $(x_2, y_2)$ ; $(x_3, y_3)$ are collinear if and only if :

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \_\_\_\_\_\_\_\_.$$

(e) The mean of the first four multiples of 3 is ________.

(f) $\underset{x \to 0}{Lt} \frac{\sin x^2}{x}$ = ________

(g) If $-\sqrt{3} - i = r(\cos\theta + i \sin\theta)$, then r = ________ , θ = ________ .

2. Prove that : **2x7=14**

(a) $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = 1$

(b) Show that the function $f(x)$ defined by

$$f(x) = x \text{ if } x > 0$$
$$= -x \text{ if } x < 0 \text{ is continuous of } x = 0$$
$$= 0 \text{ if } x = 0$$

3. Evaluate : **2x7=14**

(a) $\int \sec^4 x \tan x \; dx$

(b) $\int_1^2 \left(e^{3x} + 3x^2\right) dx$

4. (a) Find the adjoint of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$

(b) Prove that **2x7=14**

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

5. (a) Show that the function $f(x) = x^7 + 8x^5 + 1$ is increasing for all values of $x$. **2x7=14**

(b) Express $\dfrac{(6+i)(2-i)}{(4+3i)(1-2i)}$ in the form $a+ib$

6. (a) The hearts of 60 patients were examined through X-ray and the observations obtained are given below. **2x7=14**

| **Dia of heart (in mm)** | 120 | 121 | 122 | 123 | 124 | 125 |
|---|---|---|---|---|---|---|
| **No. of patients** | 7 | 9 | 15 | 12 | 6 | 11 |

Find the median.

(b) The mean weight of a class of 35 students is 45 kg. If the weight of the teachers is included the mean weight increases by 500 gms. Find the weight of the teacher.

7. (a) Find the points of local maxima and minima (if any) of the function. **2x7=14** $f(x) = (x-1)^3\ (x+1)^2$. Find also the local maximum and minimum value.

(b) If $y = e^{3\log x + 2x}$, Prove that $\dfrac{dy}{dx} = x^2(2x+3)e^{2x}$.

---