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ET-102

01152

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering)

Term-End Examination

June, 2010

ET-102 : MATHEMATICS III

Time : 3 hours *Maximum Marks : 70*

Note : *Answer **any ten** questions. All questions carry **equal** marks. Use of calculator is allowed.*

1. Show that the harmonic series of order p,

$$\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$$

is convergent for $p > 1$, and divergent for $p \leq 1$.

2. Find the fourier series for the function

$$f(x) = x \ , \quad -\pi < x < \pi,$$

when $f(x) = f(x + 2\pi)$.

3. Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$.

4. Evaluate $\oint_{\pi} \frac{1}{z-a} dz$

over any closed path enclosing the given point 'a'.

5. Find the general solution of the partial differential equation
$(x^2 - yz)\ p + (y^2 - zx)\ q = z^2 - xy.$

6. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, show that one of the values of

$$x^m y^n + \frac{1}{x^m y^n} \text{ is } 2\cos(m\theta + n\phi).$$

7. Test for convergence of the series for all possible values of x :

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \cdots$$

8. It is given that the rate of decay of radium varies as its amount present at that time. Assuming the 'half - life' of the radium to be 1600 years, find the percentage of the amount of radium disintegrated in 200 years.

9. Find the general solution of the differential equation.

$(D^2-13\ D+12)\ y=3\ e^{-2x}$

10. (a) Find the Laplace transforms of $f(t)=e^{at}$, a is real and $t \geq 0$.

(b) Find the Laplace transforms of cosh at

11. Find the inverse Laplace transforms of

$$\frac{s+2}{s^2-4s+13}.$$

12. If $w=\phi+i\,\psi$ represents the complex potential for an electric field and

$$\psi = x^2-y^2+\frac{x}{x^2+y^2},$$

determine the function ϕ.

13. Determine the poles of the function

$f(z)=\dfrac{z^2}{(z-1)^2\ (z+2)}$ and the residue at each pole. Hence evaluate $\int_C f(z)\,dz$, where C is the circle $/z/=2.5$.

14. Apply Hurwitz - Routh Criterion to determine the stability of the systems whose characteristic equations are given by :

(a) $s^4+5\ s^3+2\ s+10=0$

(b) $s^4+7\ s^3+17\ s^2+17\ s+6=0$

15. Obtain the differential equation representing this system as shown in the adjoining figure.