00832

BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

June, 2010

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : ***All*** *questions are* ***compulsory****. Use of calculator is allowed. Statistical tables are allowed.*

1. Answer ***any five*** of the following : **5x4=20**

(a) Evaluate ***any one*** of the following :

(i) Find the positive integer '*n*' so that :

$$\lim_{x\to 3} \frac{x^n - 3^n}{x-3} = 108$$

(ii) If the function f be defined as :

$$f(x) = x \text{ if } 0 < x < \frac{1}{2}$$

$$0 \text{ if } x = \frac{1}{2}$$

$$x - 1 \text{ if } \frac{1}{2} < x < 1,$$

discuss the existence of $\lim\limits_{x \to 1/2} f(x)$.

(b) Discuss the continuity of the function :

$$f(x) = \begin{cases} 2x, \text{ if } x < 2 \\ 2, \text{ if } x = 2 \\ x^2, \text{ if } x > 2 \end{cases}$$

at $x = 2$.

(c) Attempt ***any one*** of the following :

(i) Let $f(x) = x(x-1)\ (x-2),\ x \in [0,\ 2]$. Prove that f satisfies the conditions of Rolle's theorem and there is more than one c in (0, 2) such that $f'(c) = 0$.

(ii) Investigate the maximum and minimum values of :

$$x^2y^2 - 5x^2 - 8xy - 5y^2.$$

(d) Attempt ***any one*** of the following :

(i) Evaluate $\int_0^{\pi} \frac{1}{5 + 4\cos x}\, dx$

(ii) Find the area of the region in first quadrant bounded by x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

(e) Solve ***any one*** of the following :

(i) $\left(x \cos \frac{y}{x} + y \sin \frac{y}{x}\right) y$

$$= \left(y \sin \frac{y}{x} - x \cos \frac{y}{x}\right) x \frac{dy}{dx}$$

(ii) $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, $y(0) = 1$.

(f) Find the equations of the normal to the surface :

$36x^2 + 9y^2 + 5z = 72$ at (0, 2, 3).

2. Answer *any four* of the following : **4x4=16**

(a) Prove that :

$$\left(\vec{a} \times \vec{b}\right)^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{a} \end{vmatrix}$$

(b) Find the directional derivative of

$f(x,y) = \frac{x^2 + y^2}{x - y}$ at (1, 2) along the vector

$3\hat{i} + 2\hat{j} - 5\hat{k}$.

(c) A force $-5\hat{i} + 6\hat{j} - 3\hat{k}$ displaces a particle at (1, 2, 3) to the point (−1, −3, 5). Find the work done by the force.

(d) Attempt *any one* of the following :

(i) A fluid motion is given by :

$$\vec{q} = (y+z)\,\hat{i} + (z+x)\,\hat{j} + (x+y)\,\hat{k}.$$

Is this motion irrotational ?

(ii) Is the gradient of :

$$\frac{C}{|\vec{r}|}, \ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \text{ for } |\vec{r}| > 0$$

a solenoidal vector ?

(e) Let a curve C has parametric representation

as $x = \sin t, y = \sin t, \ 0 \le t \le \frac{\pi}{2}$.

Evaluate $\oint f(x, y) \, dx$,

when $f(x, y) = x^2 + y$.

(f) Attempt *any one* of the following :

(i) Use Green's Theorem to evaluate :

$$\oint_C (x - 2xy) \, dx + \left(x^2 + 3xy^2\right) dy,$$

where C is the boundary of the region
$y^2 = 8x, \ x = 2$.

(ii) If

$$\vec{F} = (x - y) \hat{k} + (y - z) \hat{j} + (z - x) \hat{i}$$

and C is the bounding curve of intersection of paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 0$, verify Stoke's Theorem.

3. Attempt *any six* parts from the following : **6x3=18**

(a) Evaluate $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$.

(b) For what values of k, the following system of equations possess a non - trivial solutions :

$$x + ky + 3z = 0$$

$$3x + ky - 2z = 0$$

$$2x + 3y - 4z = 0.$$

(c) Express the following matrix as the sum of symmetric and show symmetric matrices :

$$\begin{bmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & -1 \end{bmatrix}$$

(d) Find the inverse of the matrix :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

(e) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & -3 & -3 \\ 5 & -3 & 3 \end{bmatrix}$$

(f) Find the eigen vectors of the matrix :

$$\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

(g) Let $A = \frac{1}{3}\begin{bmatrix} x & 2 & 2 \\ 2 & 1 & y \\ 2 & z & 1 \end{bmatrix}$

If A is an orthogonal matrix, find the values of x, y and z.

4. Attempt ***any four*** of the following : **4x4=16**

(a) An insurance company used 2000 scooter drivers, 4,000 car drivers and 6,000 truck drivers. The probability of accident is 0.01, 0.03 and 0.15 respectively. One of insured persons meets an accident. What is the probability that he is a scooter driver.

(b) In a Binomial distribution with 6 independent trials, the probabilities of 3 and 4 successes are found to be 0.2457 and 0.0819 respectively. Find the parameter 'p' of the Binomial distribution.

(c) There are 500 boxes each containing 1000 ballot papers for election. The chance that a ballot paper is defective is 0.002. Assuming Poisson distribution for the number of defective ballot papers, find the number of boxes containing at least one defective ballot paper.

$\left(\text{Given } e^{-2} = 0.1353 \text{ and } e^{-3} = 0.0498\right)$

(d) For a normal distribution with variate X, the mean is 12 and the s.d. is 4. Find $P(X \geq 20)$ and $P(X \leq 12)$. (Given area under the normal fan $z = 0$ to $z = 2$ is 0.4772).

(e) A random sample of size 7 from a normal population gave a mean 977.51 and a standard derivation 4.42. Find a 95% confidence interval for the population mean. (Given $t_{0.05,6} = 1.943$, $t_{0.059,7} = 1.895$, $t_{0.025,\,6} = 2.447$, $t_{0.025,7} = 2.365$)

(f) A sample of size 20 drawn from a normal population gives a sample mean of 40 and a sample variance of 25. Test this hypothesis, that the population standard deviation is 8 at 5% level of significance. (Given X^2 at 5% level of significance $= 30.14$ for 19 d.f.).
