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MMTE–005

00549

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

December, 2010

MMTE–005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

Note : ***Question No. 1 is compulsory. Do any four questions from question number 2 to 6. Use of calculator is not allowed.***

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1. (a) Show that Hamming distance is a metric. **4**

(b) Show that in a linear code the minimum distance is equal to the minimum weight of the code. **3**

(c) Let $G = \begin{pmatrix} 0001212 \\ 0110011 \\ 1010101 \end{pmatrix}$ be a generator matrix of the ternary linear code C. Find the codewords of the code C **3**

2. (a) Find gcd $(x^6+x^5+x^4+x^3+x+1,\ x^5+x^3+x^2+x)$ in $\mathbb{F}_2[X]$. Find a(x) and b(x) such that gcd $(x^6+x^5+x^4+x^3+x+1),\ x^5+x^3+x^2+x) = a(x)\,(x^6+x^5+x^4+x^3+x+1) + b(x)\,(x^5+x^3+x^2+x)$. **5**

(b) Which of the following codes are cyclic ? 2

(i) (0000, 0101, 1010)

(ii) (00, 01, 10, 11)

Give reasons for your answer.

(c) Define a low density parity check code (LDPC) and give an example. 3

3. (a) List all the cyclic codes C_i of length 7 over $\mathbb{F}_2$ together with their generator polynomials g_i (x), their generating idempotents e_i (x) and their defining sets of each code relative to the primitive root α such that $\alpha^3 + \alpha + 1 = 0$ ie, $\alpha^3 = \alpha + 1$ 6

(b) Let C be an Reed-Solomon code over $\mathbb{F}_g$ of length $n = q-1$ and designed distance δ. Prove that C is MDS code 4

4. (a) Let C be the binary [15,5] narrow-sense BCH code of designed distance $\delta=7$, Which has defining set T= {1,2,3,4,5,6,8,9,10,12} Using primitive 15th root of unity α, such that $\alpha^4 = \alpha + 1$ and the generater polynomial $g(x) = 1+x + x^2 + x^4 + x^5 + x^8 + x^{10}$ of C. If y $(x) = x + x^4 + x^5 + x^7 + x^9 + x^{12}$ is received word, find the transmitted codeword. 6

(b) Let C be any self-dual [12 , 6, 6] ternary code. Find the weight enumerator polynomial of C. 4

5. (a) Let $f(x) = x^2 + x + 1 \in \mathbb{F}_2[x]$. Show that $f(x)$ is irreducible over $\mathbb{F}_2$. Find the elements of $\frac{\mathbb{F}_2(x)}{(f(x))}$ 2+4

(b) Show that the $\mathbb{Z}_4$ – linear codes with generator matrices 4

$$G_1 = \begin{pmatrix} 1113 \\ 0202 \\ 0022 \end{pmatrix} \text{and } G_2 = \begin{pmatrix} 1111 \\ 2002 \\ 0202 \end{pmatrix}$$

are monomially equivalent

6. (a) Define Convolutional Code and give an example 4

(b) State the Two-Way a Posteriori Probability Decoding algorithm. 6
