No. of Printed Pages : 4

MMTE–001

00509

M.Sc. (Mathematics With Applications in Computer Science) M.sc. (MACS)

Term-End Examination

December, 2010

MMTE–001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. 1 is **compulsory**. Do **any four** questions out of question No. 2 to 7. Calculators are not allowed.*

1. State whether the following statements are true or false ? Justify your answer. **2x5=10**

   (a) Any two graphs with the same degree sequence are isomorphic.

   (b) There is a 3 - regular graph with 9 vertices.

   (c) A graph in which every vertex is of even degree in Eulerian.

   (d) Any tree is 2 - colourable.

   (e) $k_{m,n}$ is Hamiltonian for all m, n ≥ 1.

2. (a) If G is a simple graph with at least two vertices, prove that G must contain two or more vertices of same degree. **2**

(b) (i) For which values of n is k_n Eulerian ? Justify your answer. 2

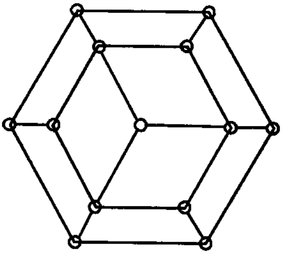
(ii) For which values of m, n is $k_{m, n}$ Eulerian ? Justify your answer.

(c) Show that any edge of a graph G is a cut - edge if and only if it belongs to no cycle. 3

(d) Show that there is a unique path between any two distinct vertices of a non-trivial tree. 3

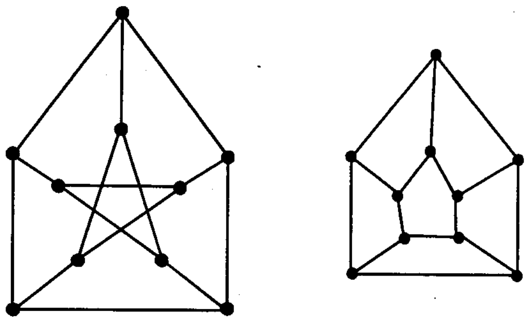
3. (a) Show that the complete graph k_n can be expressed as the union of k bipartite graphs if $n \leq 2^k$. 4

(b) Prove that, a bipartite graph with an odd number of vertices, is never Hamiltonian. Deduce that the following graph is non-Hamiltonian. 6



4. (a) Let G be a simple n-vertex graph with $\frac{n-1}{2} \leq \delta(G)$ where $\delta(G)$ is the minimum vertex degree of G. Then show that G is connected. 3

(b) Determine whether the following two graphs are isomorphic. Justify your answer. 3



(c) Does there exist a simple graph with degree sequence 4

(i) (3, 3, 5, 5, 5, 5)

(ii) (2, 3, 3, 4, 5, 5). Justify your answer.

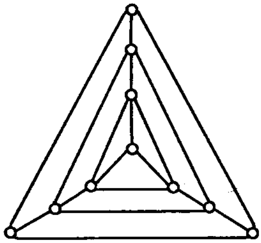
5. (a) Write all the steps in Kruskal's algorithm. 3

(b) Let $k > 0$. prove that every k - regular bipartite graph has a perfect matching. 4

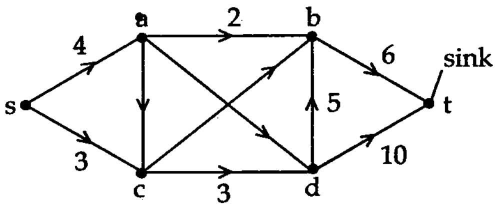
(c) Prove that every tree has at most one perfect matching. 3

6. (a) Construct a graph G for which $k(G) = 1$ $k'(G) = 2$ and $\delta(G) = 3$. Justify your choice of G. 3

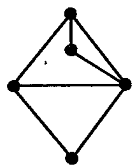
(b) Find the chromatic number of the following graph. 3



(c) In the network given below, find a maximum flow from s to t 4



7. (a) Find the dual of the following graph. Justify your answer. 4



(b) Find all self complementary graphs having 5 vertices. Justify your answer 4

(c) Let G be a graph with n vertices and e edges. Let $\delta(G)$, $\Delta(G)$ be the minimum and maximum degree of G respectively. Prove that $\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$. 2