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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

00788

Term-End Examination

December, 2010

MMT-009 : MATHEMATICAL MODELLING

Time : 1½ hours *Maximum Marks : 25*

Note : *Do **any five** questions. Use of calculator is **not** allowed.*

1. (a) In a population of birds, the proportionate birth rate and death rate are both constants, being 0.45 and 0.65 per year, respectively. Immigration occur at constant rate of 2000 birds and emigration at constant rate of 1000 birds per year. Use this assumption to formulate a model of the population. Solve the model and describe the long term behaviour of the population in two cases when the initial population is 3000 and when it is 8000. **2**

   (b) Following is the data for number of years students studied a subject and the score he/she received in that subject. **3**

Number of years	Test score
3	57
4	78
4	72
2	58
5	89
3	63
4	73
5	84
3	75
2	48

Fit the least square line to this data. What is the score of the student who has studied for two years/according to this line ?

2. (a) Indifference curves of an investor cannot intersect. Is this statement true ? Give reasons for your answer. 2

(b) A model for insect population leads to the difference equation 3

$$N_{R+1} = \frac{\lambda N_R}{1 + a N_R}$$

Where λ and a are positive constants.

(i) Write the equation in the form $N_{R+1} = N_R + R(N_R)\ N_R$ and hence identify the growth rate.

(ii) Plot the graph of $R(N_R)$ as a function of N_R.

(iii) Express the intrinsic growth rate, r and the carrying capacity, K, for this model, in terms of the parameters a and λ.

3. Do the stability analysis of one of the equilibrium solution of the following competing species system of equations with diffusion and advection 5

$$\frac{\partial N_1}{\partial t} = a_1 N1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} - V_1 \frac{\partial N_1}{\partial x}$$

$$\frac{\partial N_2}{\partial t} = -d_1 N_2 + C_1 N_1 N_2 + D_1 \frac{\partial^2 N_2}{\partial x^2} - V_2 \frac{\partial N_2}{\partial x}, 0 \le x \le 2$$

where V_1 and V_2 are constant advection velocities in x direction of the two populations with densities N_1 and N_2, respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, C_1 is the conversion rate. D_1 and D_2 are diffusion constants. The initial and boundary conditions are :

$Ni(x, 0) = fi(x) > 0,\ 0 \le x \le L,\ i = 1, 2.....$

$Ni = \overline{Ni}$ at $x = 0$ and $x = L \forall -t,\ i = 1, 2.....$

where $\overline{Ni}$ are the equilibrium solutions of the given system of equations. Also write the limitations of this model.

4. (a) A major event in a city like the common wealth games requires an unusual preparedness in terms of traffic flow. What, in your opinion, are the essentials to be considered for an effective traffic model ? List four essentials. 2

(b) Calculate the expected return and risk of a security given the following information : 3

Probabilities	0.15	0.20	0.40	0.10	0.15
Possible returns	0.20	0.16	0.12	0.05	– 0.05

5. Explain each of the following with examples : 5

(a) Reaction - diffusion model versus Advection - reaction - diffusion model.

(b) Variational matrix or Jacobian of a system of n differential equations.

(c) Ziapunahou function.

(d) Hurwitz criteria.

(e) Multiple linear regression model with k predictors.

6. In a tool crib, mechanics arrive in the morning to draw the tools. Let us assume, that the number of mechnics arrive at the rate of one per minute during first half an hour of the day/shift and the average number of mechanics served by tool crib operator is 1.2 per minute. Find out whether it is advisable to increase the number of tool crib operators. Tool crib operators are paid Rs.16 per day (shift) and mechanics Rs.32 per day. 5