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MMT-008

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2010

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 8 is **compulsory**. Answer any six questions from question number 1 to 7. Use of calculator is **not** allowed.*

1. (a) Find the matrix of the following quadratic form. **5**

$$8x_1^2+7x_2^2+3x_3^2-12x_1x_2+4x_1x_3-8x_2x_3.$$

And hence identify definiteness of the quadratic form.

(b) Joint p.m.f. of (X, Y) is given in the following table : **10**

x \ y	1	2	3
-1	.15	.20	.15
0	.12	.18	.10
1	.03	.02	.05

(i) Obtain marginal distributions of both random variables X and Y.

(ii) Test the independence of X and Y.

(iii) Obtain the conditional distribution of X given Y=2.

(iv) Obtain cov (X, Y).

2. (a) In a city 15% of the children were suffering from T.B. An investigation gives positive result in 90% cases if performed upon children affected from T.B. and gives positive result 20% times if performed upon children not affected from T.B. A child was chosen at random from the city and investigation for T.B. was made. What is the probability that result will be positive ? If it is found that result was positive, what is the probability that the child was suffering from T.B ? 6

(b) On an average 48 trains arrive at the platform of a railway station per day. The stoppage period of a train at the platform is exponentially distributed with mean 20 minutes. Obtain the following : 9

(i) average number of trains waiting in queue for the platform,

(ii) probability that more than 2 trains will be waiting in queue at any time,

(iii) average waiting time in queue and

(iv) probability that a train will wait in queue more than 10 minutes.

3. (a) Find principal components and proportions of total population variance explained by each component when the covariance matrix is given by : 6

$$\Sigma = \begin{bmatrix} 5 & 2 \\ 2 & 7 \end{bmatrix}$$

(b) Let $X \sim N_4(\mu, \Sigma)$ with 9

$$\mu = \begin{bmatrix} 2 \\ 4 \\ -1 \\ 3 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 4 & 2 & -3 & 4 \\ 2 & 4 & 0 & 1 \\ -3 & 0 & 4 & -2 \\ 4 & 1 & -2 & 8 \end{bmatrix}$$

Again Y and Z be two partitioned sub vectors of X such that $Y' = [X_1, X_2]$ and $Z' = [X_3, X_4]$.

Find :

(i) $E(Y/Z)$

(ii) $Cov\,(Y/Z)$

(iii) $r_{12.34}$

4. (a) The transition probability matrix P of a Markov Chain with states Sunny (S), Cloudy (C) and Rainy (R) in a simple weather model is given below : 5

$$P = \begin{array}{c} \\ S \\ C \\ R \end{array} \begin{array}{c} \begin{array}{ccc} S & C & R \end{array} \\ \begin{bmatrix} .7 & .2 & .1 \\ .2 & .5 & .3 \\ .1 & .5 & .4 \end{bmatrix} \end{array}$$

Also the initial probability distribution of the states is (.5, .3, .2) :

(i) What is the probability that starting from initial day all the three successive days will be cloudy.

(ii) Obtain probability distribution of the weather on second day.

(b) A radioactive source emits particles at a rate of 5 per minute according to Poisson law. Each particle emitted has probability .6 of being recorded. 5

What is the probability that in 4 minutes 10 particles will be recorded ? What is the mean and variance of number of particles recorded ?

(c) Let the lifetimes X_1, X_2 --- are i.i.d. exponential random variables with parameter $\lambda > 0$. Let $T > 0$ and age replacement policy is to be employed. 5

(i) Find μ^T.

(ii) If each replacement cost $C_1 = 3$ and extra cost $C_2 = 4$, then find the long run average cost per unit time.

5. (a) Define renewal process. Give an example to explain it. Let interoccurence times in a renewal process follows exponential distribution with rate $\lambda > 0$. What will be distribution of number of renewals in time t ? Obtain renewal function and its Laplace transform. 6

(b) Find all stationary distributions for a Markov Chain having the following transition probability matrix : 4

$$P = \begin{bmatrix} .4 & .6 \\ .7 & .3 \end{bmatrix}$$

(c) Define conjoint analysis, with its potential applications. Also, write two advantages of conjoint analysis. 5

6. (a) Income (X_1) and expenditure (X_2) obtained from a random sample of 5 families are presented in the following table. Test the hypothesis $H_0 : \mu = [9, 7]$ against $H_1 : \mu' \neq [9, 7]$ at 5% level of significance : 8

Family No.	Income (Rs. 000)	Expenditure (Rs. 000)
1	4	8
2	12	9
3	14	12
4	10	8
5	10	3

you may like to use the following values. $F_{2}, 3(.05) = 19.16$ and $F_{2}, 5(.05) = 19.30$.

(b) In a branching process, the offspring distribution (p_K) is given below : 7

$p_K = pq^K$; $q = 1 - p$, $0 < p < 1$, $K = 0, 1, 2,$ ..... What will be the probability of extinction in this branching process ?

7. (a) Suppose $n_1 = 10$ and $n_2 = 15$ observations are made on two variables X_1 and X_2 respectively. 9

Let $X_1 \sim N_2(\mu^{(1)}, \Sigma)$ and $X_2 \sim N_2(\mu^{(2)}, \Sigma)$

and $\mu^{(1)} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\mu^{(2)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 1.5 & 2 \\ 2 & 4 \end{bmatrix}$

Considering equal cost and equal prior probabilities, classify the observation [1.5, 1] in one of the two populations.

(b) A barber-stop has two barbers. Customers arrive at a rate of 5 per hour in a Poisson fashion and service time of each barber takes on average 15 minutes according to exponential distribution. The shop has 4 chairs for waiting customers. When a customer arrives in the shop and does not find an empty chair, he will leave the shop. What is expected number of customers in the shop ? What is the probability that a customer will leave the shop finding no empty chair to wait ? 6

8. State whether the following statements are true or false. Justify your answer with valid reasons. **10**

(a) The matrix given by

$$\begin{bmatrix} .2 & .7 & .1 \\ .5 & .3 & .1 \\ .4 & .4 & .2 \end{bmatrix}$$

is not a transition probability matrix of a Markov Chain.

(b) If A C B, then $P(A \mid B) < P(A)$.

(c) Every variance-covariance matrix is a non-negative definite matrix.

(d) In linear regression model the extent of fit is measured by partial correlation coefficient.

(e) In a renewal process

$P[N(+) \geq n] > P[Sn \leq t]$.