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MMT–007

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2010

MMT–007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is* ***compulsory.*** *Do any* ***four*** *questions out of question nos.* ***2*** *to* ***7****. All computations may be kept to* ***3*** *decimal places. Use of calculators is* ***not*** *allowed.*

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1. State whether the following statements are ***true*** or ***false.*** Justify your answer with the help of a short proof or a counter example : **2x5=10**

   (a) The interval of absolute stability for 3^{rd} order Taylor series method for initial value problem.

   $$y' = \lambda y,\ y(x_0) = y_0,$$

   is $]-2.78, o[$.

   (b) If Hn is a Hermite polynomial of degree n, then

   $$H_n^1 = 4n\ (n-1)\ H_{n-2}.$$

(c) The complementary function of the equation $(3x+2)^2 \, y'' + 3(3x+2) \, y' - 36y = 3x^2 + 4x + 1$ is given by $C_1 \, (3x+2)^3 + \dfrac{C_2}{(3x+2)^3}$, C_1 and C_2 are constants.

(d) The Fourier transform of the function

$$f(x) = \begin{cases} 2, & -p < x < 0 \\ 3, & 0 < x < p \\ 0 & \text{otherwise} \end{cases}$$

is $\dfrac{1}{\{\alpha} \left[-1 + 2\, e^{i\alpha p} - 3\, e^{-i\alpha p} \right]$.

(e) In the Crank-Nicolson method, the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

is replaced by the finite - difference equation

$$(1+r) \, U_i^{j+1} = U_i^j + \frac{1}{2} r \left(U_{i-1}^{j+1} + U_{i+1}^j + U_{i+1}^{j+1} + U_{i-1}^j - 4 U_i^j \right),$$

where $r = k/h^2$ and k and h are mesh spaces in dimension of t and x respectively.

2. (a) Construct Green's function for the boundary value problem 5

$$\frac{d^2 y}{dx^2} + 4y = 0, \; y(0) = y(1) = 0.$$

(b) Show that 2

$$h^{-1} \left\{ \frac{1}{(s-a)^{n+1}} \right\} = \frac{t^n}{\underline{|n}} \, e^{at}$$

(c) Prove that 3

$$J_{3/2}(x) = \sqrt{\frac{2}{(\pi x)}} \left[\frac{1}{x} \sin x - \cos x \right].$$

3. (a) Find the series solution about $x=0$ of the equation 5

$$\left(x^2 - x\right) \frac{d^2 y}{dx^2} + 3\frac{dy}{dx} - 2y = 0$$

(b) Find the solution of the heat equation 5

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the conditions

$u(x, 0)=0$, $u(0, t)=0$ and $u(1, t)=t$, using implicit Crank-Nicoloson method of finite differences with $h=\frac{1}{2}$ and $k=\frac{1}{8}$. Integrate for two time levels.

4. (a) Solve the boundary value problem 4

$$\frac{d^2 y}{dx^2} = y$$

with $y'(0)=0$ and $y(1)=1$, using second order finite difference method with $h=\frac{1}{2}$.

(b) Evaluate $L\{t^2 \sin 5t\}$. 2

(c) Solve $(x^2D^2 + 4xD + 2)y = e^x$. 4

5. (a) Using Laplace Transforms, solve $y'' + 2y' + 5y = e^{-t} \sin t$ given that $y(0) = 0$, $y'(0) = 1$. **3**

(b) Given $\frac{dy}{dx} = 1 + y^2$ with $y = 0$ when $x = 0$, find $y(0.2)$ and $y(0.4)$ using fourth order Runge-Kutta formula with $h = 0.2$. **4**

(c) Given the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ and the boundary conditions $u(0, t) = u(3, t) = 0$ and $u(x, 0) = x^2 (25 - x^2)$, use the explicit method to obtain the solution for $x = x_i = ih$ $(i = 0, 1, 2, 3,\ h = 1)$ and $t = jk \left(j = 0, 1, 2, 3 \quad k = \frac{1}{2} \right)$. **3**

6. (a) Solve $\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$ given that $u(0, t) = 0$, $u(5, t) = 0$, $u(x, 0) = \sin(\pi x)$, using Laplace transforms. **5**

(b) Using Taylor series expansions, show that the five point formula. **5**

$U_{i+1, j} + U_{i-1, j} + U_{i, j+1} + U_{i, j-1} - 4U_{i, j} = h^2 G_{i, j}$ for the Poisson equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = G(x, y)$ is of the order $O(h^2)$.

7. (a) Find the solution of the problem 5

$$\nabla^2 u = 0, 0 \le x \le 1, 0 \le y \le 1$$
$$u = x + y, \text{ on the boundary,}$$

using the Galerkin's method with triangular elements and one internal node $\left(h = \frac{1}{2}\right)$.

(b) Using Fourier transforms find the solution of the equation 5

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 u}{\partial t^2}$$

subject to conditions

$$u(x, 0) = f(x), \quad -\infty < x < \infty \text{ and}$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0.$$