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MMT-005

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

December, 2010

MMT-005 : COMPLEX ANALYSIS

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. 1 is **compulsory** . Attempt **any three** questions from question number **2** to **5**. Use of calculator is **not** allowed.*

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1. State giving reasons whether the following statements are ***true*** or ***false*** : 5x2

   (a) $\int_C z^2 \, dz = 0$ for any simple closed Contour C.

   (b) $f(z) = \sinh z$ is bounded in the complex plane.

   (c) $f(z) = \tan z$ has a removable singularity at $z = \pi/2$.

   (d) If $f(z)$ is an analytic function such that real part of f (z) is 1 then $f(z) = 1$.

   (e) $f(z) = \dfrac{2z-1}{2-z}$ has a unique point of maximum modulus in $D = \{z : |z| \leq 1\}$.

2. (a) Using $\epsilon - \delta$ definition of limit prove that 3

   $$\lim_{z \to \infty} \left(\frac{z+1}{z^2} \right) = 0.$$

(b) Let $f(z)$ be defined as **2**

$$f(z)=\begin{cases}\frac{(\bar{z})^2}{z} & , z \neq 0 \\ 0 & , z = 0.\end{cases}$$

Show that f'(0) does not exist.

3. (a) Find the set of all those complex numbers which satisfy the following equation : $e^z = -2$. **2**

(b) Let C denote the circle $|z| = 2$, described in the Counter-Clockwise direction. Show that **3**

$$\left|\int_C \frac{\text{Log}z}{z^2} dz\right| \leq \pi(\underline{\pi + ln z}).$$

4. (a) Let $f(z) = \frac{10z^3}{z^2(z^2+9)}$ and let **2**

C_1 denote the circle $|z| = 2$ in the counter clockwise direction and C_2 is the circle $|z| = 1$ in the clockwise direction. Then prove that $\int_{C_1} f(z)\,dz = -\int_{C_2} f(z)\,dz$

(b) Find the image of the square with vertices at $(-1+i)$, $(1+i)$, $(1-i)$ and $-(1+i)$ under the transformation $\omega = e^{\frac{i\pi}{4}}(z+1+i)$. **3**

5. Show that $\int_0^{2\pi} \frac{2d\theta}{\left(2+\sqrt{2}\sin\theta\right)} = 2\sqrt{2}\,\pi$ 5