No. of Printed Pages : 4

MMT-003

00449

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

December, 2010

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

Note : *Question No. 1 is* ***compulsory****. Also do* ***any four*** *questions from Q. No. 2 to Q. No. 6. Calculators are* ***not*** *allowed.*

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1. State which of the following statements are true and which are false. Give reasons for your answer.

(a) There exists an element of order 15 in the symmetric group S_7. **2**

(b) If $\alpha = a + ib$, $b > 0$, is a complex number, satisfying a polynomial of degree 5 over Q, then α^4 satisfies a polynomial of degree 4 over Q. **2**

(c) There exists a 3×3 real orthogonal matrix with $\left(\frac{-1}{4}, \frac{1}{2}, \frac{3}{4}\right)$ as its first row. **2**

(d) A subgroup of a group G contained in the centre of G is a normal subgroup of G. **2**

(e) A non-trivial real-valued irreducible character of a group takes both positive and negative values. **2**

2. (a) Show that the product group $\left(\mathbb{Z}/_{47\mathbb{Z}}\right)^* \times \left(\mathbb{Z}/_{61\mathbb{Z}}\right)^*$ has a cyclic subgroup of order 345, where G* denotes the group of units of G. 5

(b) If F is an algebraic extension of the field K, and D is an integral domain such that $K \leq D \leq F$, then show that D is a field. 3

(c) Let F (α) be a finite extension of F with [F(α) : F] odd. Show that $F(\alpha^2) = F(\alpha)$. 2

3. (a) Solve the following simultaneous system of congruences : 4

$$x \equiv 2 \pmod 5$$

$$x \equiv 1 \pmod 7$$

$$x \equiv 3 \pmod{11}$$

(b) Show that the union of all Sylow 3-subgroups of S_4, after removing the identity element, is a conjugacy class in S_4. 3

(c) For any prime number p, show that there are no field homomorphisms between $\mathbf{F}_{p^2}$ and $\mathbf{F}_{p^3}$ in any direction. 3

4. (a) If χ is a character of a finite-dimensional representation of a finite group G, show that $|\chi(g)|$ is maximum for g = identity element of G. 5

(b) A stamp automaton s has a capacity of seven stamps. Define this as a semi-automaton such that the state s_i means there are i stamps in s. The inputs are 'no coin inserted', 'a correct coin inserted', 'a wrong coin inserted'. Describe this semi-automaton by a table. Further, extend it to an automaton by taking the outputs as 'no output', 'a stamp', 'a coin'. Describe the automaton by a table. 5

5. (a) Let P be a matrix in $SO_3(C)$. Show that 1 is an eigen value of P. 4

(b) Let $F \subset K \subset L$ be a tower of fields. Show that, if L is algebraic over K and K is algebraic over F, then L is algebraic over F. 4

(c) Determine the number of non-isomorphic abelian groups of order 60. 2

6. (a) Write the class equation of a group G of order n. Is there a group of order 10 with class equation $1+2+2+5$? If yes, exhibit the group and its conjugacy classes. If no, give reasons for your answer. 3

(b) Find the missing row(s) in the character table below : **5**

	(1) 1	(3) a	(6) (b)	(6) (c)	(8) d
χ_1	1	1	1	1	1
χ_2	1	1	-1	-1	1
χ_3	3	-1	1	-1	0
χ_4	3	-1	-1	1	0

(c) Obtain the prime subfield of a field extension of $R(x)$. **2**