

**BACHELOR OF COMPUTER
APPLICATIONS (BCA) (REVISED)**

Term-End Examination

June, 2024

BCS-012 : BASIC MATHEMATICS

Time : 3 Hours

Maximum Marks : 100

***Note :** Question No. 1 is compulsory. Attempt any **three** questions from the remaining questions.*

1. (a) Show that : 5

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(b) If : 5

$$X = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \text{ and } I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

find $(X - I_2)^2$.

- (c) Find the sum upto n terms of the series : 5

$$0.3 + 0.33 + 0.333 + \dots$$

- (d) If 7 times the 7th term of an A. P. is equal to 11 times the 11th term of the A. P., find the 18th term of A.P. 5

- (e) If $1, \omega, \omega^2$ are the cube roots of unity, find : 5

$$(2 + \omega + \omega^2)^6 + (3 + \omega + \omega^2)^6$$

- (f) Find the quadratic equations with real coefficients and with the following pair of roots : 5

$$\left(\frac{m-n}{m+n}\right), -\left(\frac{m+n}{m-n}\right)$$

- (g) Evaluate : 5

$$\int x\sqrt{3-2x} dx$$

- (h) A spherical balloon is being inflated at the rate of 900 cubic centimetres per second. How fast is the radius of the balloon increasing when the initial radius is 25 cm ? 5

2. (a) Solve the following set of equations by using the matrix method : 5

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y - 5z = 9$$

- (b) Use the principle of mathematical induction to show that : 5

$$1 + 4 + 7 + \dots + (3n - 2) = \frac{1}{2}n(3n - 1)$$

- (c) Find the quadratic equation with real coefficients and with the pair of roots : 5

$$\frac{1}{5 - \sqrt{72}}, \frac{1}{5 + 6\sqrt{2}}$$

- (d) If :

$$\left(\frac{1-i}{1+i} \right)^{10} = a + ib,$$

then show that $a = 1$ and $b = 0$. 5

3. (a) If :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 5 & 2 \\ 2 & -1 & 7 \end{bmatrix},$$

show that A is row equivalent to I_3 . 5

(b) Solve the inequality :

$$\frac{2x-5}{x+2} < 5, \quad x \in \mathbb{R}$$

Also graph the solution set. 5

(c) Solve the equation :

$$32x^3 - 48x^2 + 22x - 3 = 0$$

given the roots are in A.P. 5

(d) Determine the point of local maxima and local minima of : 5

$$f(x) = x^3 - 6x^2 + 9x + 100$$

4. (a) Check the continuity of the function $f(x)$ given below at $x = 0$: 5

$$f(x) = \begin{cases} \frac{2|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

- (b) Determine the vector and the cartesian equations of the line passing through the point $(1, -1, -2)$ and parallel to the vector : 5

$$3\hat{i} - 2\hat{j} + 5\hat{k}$$

- (c) Determine the length of the curve $y = 2x^{3/2}$ from point $(1, 2)$ to $(4, 16)$. 5
- (d) Find the sum of all integers between 100 and 1000 that are divisible by 9. 5

5. (a) Find the maximum value of $5x + 2y$ subject to the following constraints : 5

$$-2x - 3y \leq -6$$

$$x - 2y \leq 2$$

$$6x + 4y \leq 24$$

$$-3x + 2y \leq 3$$

$$x \geq 0, y \geq 0.$$

- (b) Find the area bounded by the curves $y = x^2$ and $y^2 = x$. 5

(c) Reduce the matrix :

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

to its normal form and hence determine its rank. 5

(d) If :

$$\vec{a} + \vec{b} + \vec{c} = 0$$

show that :

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

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