BME-001

01440

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

June, 2015

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of calculator is allowed.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Solve the differential equation :

$$(2x - 3y + 2)dx + 3(4x - 6y - 1)dy = 0$$

(b) Find the area of the regions bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

(c) Calculate :

(i) $\int \sin^3 x \cos^2 x \, dx$

(ii) $\int x^2 \cos x \, dx$

(d) Calculate the maximum and minimum of the function

$$f(x) = x^5 - 5x^4 + 5x^3 - 1$$

with help of first derivative test.

(e) Find the points where the tangent to the circle $x^2 + y^2 = 25$ is parallel to the line $2x - y + 6 = 0$.

(f) If

$$f(x) = \begin{cases} 1, & \text{if} \quad x \leq 3 \\ ax + b, & \text{if} \quad 3 < x < 5 \\ 7, & \text{if} \quad 5 \leq x \end{cases}$$

determine the value of a and b so that f(x) is continuous.

2. Answer any ***four*** of the following questions : *4×5=20*

(a) Use Green's theorem to evaluate $\oint_C \left[(x^2 + xy)\, dx + (x^2 + y^2)\, dy\right]$, where C is the boundary of the square $y = \pm 1$, $x = \pm 1$.

(b) Show that the vector A

$A = (x + 3y)\, \hat{i} + (y - 3z)\, \hat{j} + (x - 2z)\, \hat{k}$ is solenoidal.

(c) Find curl F, where $F = \nabla (x^3 + y^3 + z^3 - 3xyz)$.

(d) Find the directional derivative $\frac{\partial f}{\partial s}$, for $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point P (2, 1, 3) in the direction of the vector $a = \hat{i} - 2\hat{k}$.

(e) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the points (1, 2, –1).

(f) Evaluate :

$$\oint_C \left[(x^2 - y)\, dx + (y^2 - z)\, dy + (z^2 - x)\, dz\right],$$

where C is the curve from origin O to the point A (1, 1, 1) along the straight line OA.

3. Attempt any ***five*** of the following questions : *5×3=15*

(a) Use Cramer's Rule to solve the system

$$x + 2y - z = 2,$$

$$2x + y - 2z = -2,$$

$$3x - y + 2z = 7.$$

(b) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}.$$

Show that the equation is satisfied by A and hence obtain the inverse of A.

(c) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}.$$

(d) For what value of a and b, does the system of equations

$$2x + ay + 6z = 8$$

$$x + 2y + bz = 5$$

$$x + y + 3z = 4$$

have

(i) a unique solution ?

(ii) infinitely many solutions ?

(e) By using elementary row transformation find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}.$$

(f) Use the Gauss elimination to solve the following system of linear equations :

$$x - y + z + t = 6$$

$$y + 2z - 2t = 4$$

$$x - y + 2z = 8$$

$$x + 3z - t = 10$$

(g) Identify whether the set is linearly independent or not :

$$S = \{(1, 1, 1), (2, 1, 1), (1, 2, 2)\}$$

4. Answer any ***three*** of the following questions : *3×5=15*

(a) A part time student is taking two courses namely, Statistics and Finance. The probability that the student will pass the Statistics course is 0·60 and the probability of passing the Finance course is 0·70. Find the probability that the student

(i) will pass in least one course,

(ii) will fail in both the courses.

(b) An anti-aircraft gun can take a maximum of four shots on the enemy's plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0·4, 0·3, 0·2 and 0·1 respectively. Find the probability that the gun hits the plane.

(c) Three urns A, B and C contain 6 red and 4 black balls, 2 red and 6 black balls and 1 red and 8 black balls respectively. An urn is chosen at random and a ball is drawn from the urn. If the ball drawn is red, find the probability that the ball was drawn from urn A.

(d) Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random

(i) exactly two will be defective, and

(ii) more than one will be defective.

(e) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random (i) 1, (ii) 0, (iii) at most 2 bolts will be defective.