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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

June, 2015

01061

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Question No. 1 is* ***compulsory****. Attempt any other* ***eight*** *questions from Q.No. 2 to Q.No. 15. Use of calculator is allowed.*

1. Fill in the blanks. All questions are compulsory. *7×2=14*

   (a) The series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots$ is _______ .

   (b) By D'Alembert's test if Σu_n is a positive term series such that $\lim\limits_{n \to \infty} \frac{u_{n+1}}{u_n} = l$, then the series diverges if _______ .

   (c) If f(x) is an odd function on $(-\pi, \pi)$, then for Fourier Series a_n = ________ .

   (d) If $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x alone, then the Integrating Factor is ________ .

(e) For the differential equation

$$\frac{d^4y}{dx^4} - a^4y = 0,$$

the solution of the equation is ________ .

(f) The Laplace transform of $\{e^{3t}, u(t - 2)\}$ is ________ .

(g) The poles of the function $\dfrac{(z+2)}{(z+1)^2\,(z-2)}$ are ________ .

2. (a) Discuss the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(4n^2-1)}$. $3\frac{1}{2}$

(b) Prove that the series

$$\frac{1}{2}\frac{x^3}{3} + \frac{1}{2.4}\frac{x^5}{5} + \frac{1.3}{2.4.6}\frac{x^7}{7} + \ldots$$

is convergent when $0 < x \leq 1$. $3\frac{1}{2}$

3. Find the Fourier series for

$$f(x) = \begin{cases} 0, & \text{for } -\pi \leq x \leq 0 \\ x, & \text{for } 0 \leq x \leq \pi \end{cases}$$

and prove that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$ 7

4. Show that in the interval (0, 1) the Fourier series

$$\cos \pi x = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{n}{4n^2-1} \sin 2n\pi x.$$ 7

5. (a) Discuss if $f(z) = z\bar{z}$ is analytic. 3

(b) Find Taylor series of $f(z) = \frac{1}{z}$, about $z = -1$ and $z = 1$. 4

6. Evaluate $I = \int_0^{2\pi} \frac{d\theta}{a + b \sin \theta}$, a, b real, $|b| < |a|$. 7

7. (a) Find the bilinear transformation that maps the points $-i, 0, i$ into the points $-1, i, 1$. 3

(b) Prove that $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx = \frac{5\pi}{12}$. 4

8. Solve the differential equation

$(D^2 + 2)y = x^3 + e^{-2x} + \cos 3x$, where $D \to \frac{d}{dx}$. 7

9. If $P_m(x)$ and $P_n(x)$ are Legendre polynomials, then prove that

$$\int_{-1}^{1} P_m(x) P_n(x) \, dx = 0, \text{ if } m \neq n$$

$$= \frac{2}{(2n+1)}, \text{ if } m = n.$$ 7

10. Solve the P.D.E. of

$(D_x^2 - 6D_xD_y + 9D_y^2)z = 12x^2 + 36xy$,

where $D_x = \frac{\partial}{\partial x}$ and $D_y = \frac{\partial}{\partial y}$. 7

11. Let a rod of length 20 cm be initially at uniform temperature of 25°C. Suppose that at time t = 0 the end x = 0 is cooled to 0°C while the end x = 20 is heated to 60°C and both are thereafter maintained at those temperatures. Find the temperature distribution in the rod at any time t. 7

12. Using Convolution theorem evaluate

$$L^{-1}\left\{\frac{16}{(s-2)\,(s+2)^2}\right\}. \qquad 7$$

13. Using Laplace Transform solve the differential equation

$$y'' + y = 2e^t, \;\; y(0) = 0, \;\; y'(0) = 2. \qquad 7$$

14. Find the characteristic function, transfer function, frequency response function and characteristic roots of the equation $(D^3 + 1)x = f$, where $D \to \frac{d}{dx}$. 7

15. A series circuit in which both the charge and the current are initially zero contains the element L = 1 H, R = 1000 Ω, C = $6{\cdot}28 \times 10^{-6}$ F. If a constant voltage E = 24 V is suddenly switched into the circuit, find the peak value of the resultant current. 7