| MCS-033 |

MCA (Revised)
Term-End Examination
06863   June, 2015

MCS-033 : ADVANCED DISCRETE MATHEMATICS

Time : 2 hours   *Maximum Marks : 50*

Note : *Question number **1** is **compulsory**. Attempt any* ***three*** *questions from the rest.*

1. (a) Explain divide and conquer relations with an example.   *4*

    (b) Find the order and degree of each of the following recurrences :   *4*

    (i) $a_n = a_{n-1} + a_{n-2}$

    (ii) $a_n = a_{n-1} + n$

    (iii) $a_n = n\, a_{n-1} + (-1)^n$

    (iv) $a_n = a_n\, a_0 + a_{n-1} + ... + a_0\, a_n (n \geq 2)$

    (c) Explain generating functions with suitable examples.   *4*

(d) Define Graph and Subgraph. Give an example of a subgraph H of a graph G with $\delta(G) < \delta(H)$ and $\Delta(H) < \Delta(G)$. **4**

(e) Define Tree and Bipartite graph. Is tree a bipartite graph ? Justify your answer. **4**

2. (a) What are Hamiltonian graphs ? Construct a non-Hamiltonian graph on 5-vertices. **5**

(b) Show that K_5 is not planar. **5**

3. (a) What is the chromatic number of the following :

(i) A tree with at least two vertices **2**

(ii) An even cycle C_{2n}, $n \geq 2$ **2**

(iii) An odd cycle C_{2n+1}, $n \geq 1$ **2**

(b) State and prove Euler's formula. **4**

4. (a) Find the sum of the series given as,

$$\sum_{k=0}^{\infty} \frac{(k+1)^2}{\lfloor k} = \frac{1^2}{\lfloor 0} + \frac{2^2}{\lfloor 1} + \dots + \frac{(n+1)^2}{\lfloor n} + \dots,$$

using exponential generating functions. **5**

(b) How many integer solutions are there to $a_1 + a_2 + a_3 + a_4 + a_5 = 28$, where $a_k > k$ for each k, where $1 \leq k \leq 5$? **5**

5. (a) Solve the recurrence $a_n = 4a_{n-2}$, where

    (i) $a_0 = 4$, $a_1 = 6$ *2*

    (ii) $a_0 = 6$, $a_2 = 20$ *2*

    (iii) $a_1 = 6$, $a_2 = 20$ *2*

(b) Using an appropriate substitution, solve the recurrence given by,

$$y_n = \left(\frac{n-1}{n}\right) y_{n-1} + \frac{1}{n}, \quad n \geq 1,$$

where $y_0 = 5$.   *4*