No. of Printed Pages : 6 **BCS–012**

BACHELOR OF COMPUTER APPLICATIONS (BCA) (REVISED)

Term-End Examination

December, 2023

BCS-012 : BASIC MATHEMATICS

Time : 3 Hours *Maximum Marks : 100*

Note : *Question Number **1** is compulsory. Attempt any **three** questions from the remaining questions.*

1. (a) Show that : 5

$$\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0\,.$$

(b) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$, show that $f(A) = O_{2\times 2}$. Use this result to find A^5. 5

(c) Show that 7 divides $2^{3n} - 1 \ \forall n \in N$. 5

(d) If $1, \omega, \omega^2$ are cube roots of unity, show that : 5

$$(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega^4)(1+\omega^6)(1+\omega^8) = 4$$

(e) If $y = ae^{mx} + be^{-mx} + 4$, show that : 5

$$\frac{d^2y}{dx^2} = m^2(y-4).$$

(f) If α, β are roots of $x^2 - 2kx + k^2 - 1 = 0$ and $\alpha^2 + \beta^2 = 10$, find k. 5

(g) Find the value of λ for which the vectors : 5

$$\vec{a} = 2\hat{i} - 4\hat{j} + 3\hat{k},$$

$$\vec{b} = \lambda\hat{i} - 2\hat{j} + \hat{k}$$

and $\vec{c} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

are coplanar.

(h) Find the angle between the pair of lines : 5

$$\frac{x-5}{2} = \frac{y-3}{3} = \frac{z-1}{-3}$$

and $$\frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-3}.$$

2. (a) Solve the following set of linear equations by using matrix inverse : 5

$$3x + 4y + 7z = -2$$

$$2x - y + 3z = 6$$

$$2x + 2y - 3z = 0.$$

(b) Use the principle of mathematical induction to prove that : 5

$$1^3 + 2^3 + ... + n^3 = \frac{1}{4} n^2 (n+1)^2,$$

for every natural number n.

(c) Find how many terms of the GP $\sqrt{3}, 3, 3\sqrt{3}$, ... add up to $120 + 40\sqrt{3}$. 5

(d) Write De Moivre's theorem and use it to find $(i + \sqrt{3})^3$. 5

3. (a) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 5 & 7 \\ 5 & 3 & 4 \end{bmatrix}$, show that A(adj A) = 0. 5

(b) Solve the inequality $\left|\frac{x-4}{2}\right| \leq \frac{5}{12}$ and graph the solution set. 5

(c) Solve the equation $8x^3 - 14x^2 + 7x - 1 = 0$, given that roots are in GP. 5

(d) Verify that $f(x) = 1 + x^2 \ln\left(\frac{1}{x}\right)$ has a local maxima at $x = \frac{1}{\sqrt{e}}, (x > 0)$. 5

4. (a) Evaluate : 5

$$\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1-2x}}{x}.$$

(b) Find the shortest distance between the lines : 5

$$\vec{r_1} = (3\hat{i} + 4\hat{j} - 2\hat{k}) + t(-\hat{i} + 2\hat{j} + \hat{k})$$

and $\vec{r_2} = (\hat{i} - 7\hat{j} - 2\hat{k}) + t(\hat{i} + 3\hat{j} + 2\hat{k})$.

(c) Determine the length of curve $y = \frac{2}{3}x^{\frac{3}{2}}$ from (0, 0) to $\left(1, \frac{2}{3}\right)$. 5

(d) Find the sum of all the integers between 100 and 1000 that are divisible by 7. 5

5. (a) Determine the area between the two curves $y = 3 + 2x, y = 3 - x, 0 \le x \le 3$ using integration. 5

(b) Find the direction cosines of the lines passing through the two points (1, 2, 3) and (–1, 1, 0). 5

(c) Find the maximum value of $2a + 5b$ subject to the following constraints : 5

$$-3a - 2b \le -6$$

$$-2a + b \le 2$$

$$4a + 6b \le 24$$

$$2a - 3b \le 3$$

$$a \ge 0 \text{ and } b \ge 0.$$

(d) Reduce the matrix $A = \begin{bmatrix} 5 & 3 & 8 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ to normal form and hence find its rank. 5