POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

June, 2024

MST-004 : STATISTICAL INFERENCE

Time : 3 Hours   *Maximum Marks : 50*

Note : *Question no.* ***1*** *is compulsory. Attempt any* ***four*** *questions from question* ***2*** *to* ***7***. *Scientific (non-programmable) calculator is allowed. Formulae and statistical tables booklet for PGDAST is allowed. Symbols have their usual meanings.*

1. State whether the following statements are TRUE or FALSE. Give reasons in support of your answers :   2×5=10

   (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ

and known variance $\sigma^2 = 5$. Then, $H_0 : \mu = 47$ is not a composite hypothesis.

(b) If $H_0 : \theta \geq \theta_0$ and $H_1 : \theta < \theta_0$, then to test H_0, the test well be right tailed test, having critical region under right tail only.

(c) Diameter of a steel ball bearing produced by a semi-automatic machine is known to be distributed normally with mean 12 cm and standard deviation 0.1 cm. If we take a random sample of size 25 with replacement then standard error of sample mean will be 0.002.

(d) Standard deviation of a Chi-square distribution is 4, then degree of freedom will be 2.

(e) In Chi-square test for independence of attributes suppose we have two characteristics A and B. Suppose characteristics A and B have r and s categories, respectively. If $r = 4$ and degree of freedom of the test is 24, then $s = 6$.

2. (a) A soft-drink vending machine is set so that the amount of drink dispensed is a random variable with mean 200 milliliters and standard deviation 15 milliliters. What is the probability that the average (mean) amount dispensed in a random sample of size 36 is at least 204 milliliters.   5

   (b) In a large city A, 800 persons out of a sample of 1000 persons were found to be alcohol drinkers. In another large city B, 800 persons were alcohol drinkers in a sample of 1200 persons. Construct 95% confidence interval for the difference in proportions of the alcohol drinkers of the cities A and B.   5

3. (a) Let X_1 and X_2 be independent and identical random variables with mean μ and variable σ^2. Suppose that we have two estimator of μ :

$$\hat{\theta}_1 = \frac{X_1 + X_2}{2}, \hat{\theta}_2 = \frac{X_1 + 3X_2}{4}$$

   (i) Arc $\hat{\theta}_1$ and $\hat{\theta}_2$ unbiased estimators of μ ?   2

   (ii) Find the variance of each estimator.   2

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(b) Suppose $X_1, X_2, X_3, ..., X_n$ are indepen-dent Poisson random variables each having mean λ. Determine the maximum likelihood estimator of λ.   6

4. (a) Suppose the time taken by students in completing a task is assumed to be normally distributed with mean μ and variance σ^2. The time taken (in minutes) by 21 students in completing the task is recorded as :

15, 18.5, 12, 15, 17, 19, 17.5, 16.5, 10.5, 9, 21, 17.8, 18.2, 19.5, 11.1, 15.8, 18.2, 16.9, 12.9, 14.1, 17.7. Obtain sided 95% confidence interval for μ.   5

(b) Suppose that the manufacturer of a new drug wants to test the null hypothesis $H_0 : \theta = 0.90$ against the alternative hypothesis $H_1 : \theta = 0.60$. His test statistics is X, the observed number of successes (recoveries) in 20 trials, and he will accept the null hypothesis, if X > 14; otherwise, he will reject it. Fid α and β.   5

5. (a) A public health official claims that the mean home water use is 350 gallons a day. To verify this claim, a study of 20 randomly selected homes was instigated

and the following data of daily used water of the homes were obtained and given as :

340, 356, 332, 362, 318, 344, 386, 402, 322, 360, 362, 354, 340, 372, 338, 375, 364, 355, 324, 370

Do the data contradict the official's claim ?

5

(b) An industrial safety program was recently founded in the computer chip industry. The average weekly loss (averaged over 1 month) in labor-hours due to accidents in 10 similar plants both before and after the program are as follows :

Plant	Before	After
1	30.5	23
2	18.5	21
3	24.5	22
4	32	28.5
5	16	14.5
6	15	15.5
7	23.5	24.5
8	25.5	21
9	28	23.5
10	18	16.5

Under the normality assumption, test at the 5% level of significance, whether the

safety program has been proven to be
effective.   5

6. A survey of 320 families with 5 children each,
    revealed the following distribution :

No. of Boys	No. of Girls	No. of Families
5	0	14
4	1	56
3	2	110
2	3	88
1	4	40
0	5	12

In this result consistent with the hypothesis
that male and female births are equally
probable at 5% level of significance ?   10

7. (a) The following data give the milk
         production (in thousand kg) in full cream
         by 50 different dairies ?

         17, 15, 20, 29, 19, 19, 22, 25, 27, 9, 24, 20,
         17, 6, 24, 14, 15, 23, 24, 26, 19, 23, 28, 19,
         16, 22, 24, 17, 20, 13, 19, 23, 28, 19, 16, 22,
         24, 17, 20, 13, 19, 10, 23, 18, 31, 13, 20, 17,
         24, 14

         Test that median $(\tilde{\mu})$ production of milk in
         dairies is 21.5 thousand kg at 1% level of
         significance.   6

    (b) Differentiate between parametric and non-
         parametric tests.   4

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