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MASTER'S DEGREE PROGRAMME IN PHYSICS (MSCPH)

Term-End Examination

June, 2024

MPH–008 : QUANTUM MECHANICS—II

Time : 2 Hours *Maximum Marks : 50*

Note : *Answer any* ***five*** *questions. Symbols have their usual meanings. The marks for each question are indicated against it. You may use a calculator.*

1. (a) Show that the momentum operator in quantum mechanics is proportional to the generator of infinitesimal translations. 5

(b) What do you understand by time-reversal ? Show that the commutation relation $[\hat{x}, \hat{p}_x] = i\hbar$ is preserved under time-reversal. 5

2. Define the action of the permutation operator $\hat{P}_{12}$ on a system of two identical particles (1 and 2) and two states $| \psi_P >, | \psi_{P'} >$.

Show that $\hat{P}_{12}^2 = I$. If the states $| \psi_P >$ and $| \psi_{P'} >$ are the normalized eigen kets of a single observable O for the particles, show that $\hat{P}_{12} \hat{O}_1 \hat{P}_{12}^{-1} = \hat{O}_2$. 2+3+5

3. Write down the total angular momentum eigen kets $| j, m_j >$ for $j_1 = j_2 = \frac{1}{2} \left(\hat{J} = \hat{J}_1 + \hat{J}_2 \right)$.

Obtain the total angular momentum states in the basis of the direct product kets. 2+8

4. Consider a simple harmonic oscillator with the modified Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2(1 + \alpha)x^2$, where $\alpha << 1$. Calculate the first order perturbation correction to the ground state energy and eigen function of the simple harmonic oscillator Hamiltonian

$$H_o = \frac{p^2}{2m} + \frac{1}{2} mw^2x^2.$$

Given that for $u \neq n$:

$$< \phi_k \mid x^2 \mid \phi_n > =$$

$$\begin{cases} \frac{\hbar}{2mw} \sqrt{(n+1)(n+2)}, & \text{for } k = n + 2 \\ 0 , & \text{otherwise} \end{cases}$$

where $\mid \phi_k >, \mid \phi_n >$ are the eigenkets of the unperturbed hamiltonian. Take the ground state wave function as

$$\phi_0(x) = \left(\frac{a}{\sqrt{\pi}}\right)^{\frac{1}{2}} \exp\left(\frac{-a^2x^2}{2}\right), \text{ where } a^2 = \frac{mw}{\hbar}$$

and $\int_{-\infty}^{\infty} x^2 e^{-ax^2} = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$. 6+4

5. Use the variational method to determine the upper bound to the ground state energy of a simple harmonic oscillator with the Hamiltonian :

$$\hat{H} = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}mw^2x^2$$

using the trial wave function

$$\psi(x) = \begin{cases} N(\beta^2 - x^2), & \text{for} -\beta \le x \le \beta \\ 0, & \text{elsewhere} \end{cases}$$

where β is the variational parameter. 10

6. Consider a system described by a time-dependent Hamiltonian of the form $\hat{H} = \hat{H}_o + \hat{V}(t)$, where $\hat{H}_o$ has no explicit time-dependence defining the state vector in the interaction picture, $|\psi(t)>_I$, derive the equation for the time evolution of the state

vector in the interaction picture. What is the advantage of working with the interaction picture for a time dependent Hamiltonian ?

8+2

7. (a) For a constant perturbation turned on at $t = 0$:

$$V(t) = \begin{cases} 0, & t < 0 \\ V_0, & t \geq 0 \end{cases}$$

calculate the probability that the state $| m >$ is occupied by the system at time t, using the results of first order perturbation theory. It is given that the system is in a state $| i >$ at $t = 0$. 5

(b) Write the Dirac equation for a free particle. Show that the operator C_α, where α is the Dirac matrix can be interpreted as the velocity operator. 5

8. What is the first Born approximation ? Write the expression for the scattering amplitude for elastic scattering from a central potential in the first Born approximation. How is this modified for low energy scattering ?

Calculate the low energy scattering amplitude for : 3+1+1+5

$$V(r) = \begin{cases} V_0, & r \leq R_0 \\ 0, & r > R_0 \end{cases} .$$