No. of Printed Pages : 5 **MPH–006**

MASTER OF SCIENCE (PHYSICS)

(MSCPH)

Term-End Examination

June, 2024

MPH–006 : CLASSICAL MECHANICS—II

Time : 2 Hours *Maximum Marks : 50*

Note : *Answer any* ***five*** *questions. The marks for each question are indicated against it. Symbols have their usual meanings. You may use a calculator.*

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1. (a) Show that if the Lagrangian of a system is invariant under a uniform translation $\vec{r}_i = \vec{r}_i + \vec{\varepsilon}$, the total linear momentum of

the system is conserved. $\vec{\varepsilon}$ is the infinitesimal displacement. 5

(b) Consider a particle moving in one dimension described by Lagrangian : 5

$$L = \left(\frac{1 + 2\beta x}{2}\right)(\dot{x} - \alpha x^2)^2 - \frac{1}{2}\omega^2 x^2$$

where α, β and ω are constants. Obtain the Hamiltonian of the system.

2. Consider a relativistic free particle of mass m and energy $E = mc^2$ as measured in a given inertial frame of reference. If the Lagrangian of the free particle is given as :

$$L(\vec{r}, \vec{v}) = m\,(v^2 - c^2)$$

Write the Lagrangian of the particle in a conservative potential, $V(r)$. Show that the

Hamiltonian of the relativistic particle in the conservative potential, V(r), is : 10

$$H(\vec{r}, \vec{p}) = \sqrt{p^2c^2 + m_0^2c^4} + V(r)$$

3. Obtain Hamilton's equations of motion from the variational principle. 10

4. Obtain the generating function F(q, p) for the canonical transformation : 10

$$Q = \sqrt{q}\cos 4p, P = \sqrt{q}\sin 4p.$$

5. Consider a particle of mass m and charge q, in a uniform EM field. If the Lagrangian of the system is :

$$L(\vec{r}, \dot{\vec{r}}, t) = \frac{1}{2} m\, \dot{\vec{r}}^2 + q(\dot{\vec{r}} \cdot \vec{A}) - q\phi$$

where ϕ and $\vec{A}$ are respectively the scalar and vector potentials of the gauge transformation and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Obtain the Hamiltonian when the Lagrangian is added by a total time derivative, $q \dfrac{d\psi(\vec{r}, t)}{dt}$. Here $\psi(\vec{r}, t)$ is some twice differentiable function. 10

6. (a) Using Poisson brackets, obtain the value of α and β such that the transformation $q = \alpha\sqrt{P} \sin Q, p = \alpha\sqrt{P} \cos Q$ is canonical. 8

   (b) State Liouville's theorem and write the mathematical expression. 2

7. Consider a projectile of mass m, described by the Hamiltonian :

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) + mgz$$

where g is the strength of the uniform gravitational field.

(i) Obtain Hamilton's characteristic function, $w(z)$ and Hamilton's principal function. 7

(ii) Obtain the new coordinates Q_1, Q_2 and Q_3. 3

8. (a) Write the conditions for principal moments of inertia for a symmetric top and spherical top. 2

(b) Consider a uniform square plate of length $x = y = a$ and mass M. Obtain the moment of inertia I_{xx} about x-axis and I_{zz} about z-axis. 4+4