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MASTER'S DEGREE PROGRAMME IN PHYSICS (MSCPH)

Term-End Examination

June, 2024

MPH-004 : QUANTUM MECHANICS–I

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any* ***five*** *questions.*

(ii) Symbols have their usual meanings.

(iii) You may use a calculator.

(iv) The values of physical constant are given at the end.

(v) The marks for each question are indicated against it.

1. (a) What do you understand by wave-particle duality ? Explain why we do not observe the consequences of wave particle duality in our day-to-day experiences. Calculate the de Broglie wavelength of an electron accelerated from rest through a potential difference of 200 V. 5

(b) Use the concertainty principle to determine the approximate size of an atom. 5

2. The wave function for a particle is given by :

$$\psi(x) = \begin{cases} N(a^2 - x^2) & -a \leq x \leq a \\ 0 & \text{elsewhere} \end{cases}$$

Calculate the normalization constant N and the expectation value of the position $\langle x \rangle$. 5+5

3. Write the time-independent Schrödinger equation for a particle of mass m moving in a step potential defined by :

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ u > 0 & \text{for } x > 0 \end{cases}$$

State the boundary conditions and solve the Schrödinger equation when the energy of the particle E is less than u. Determine the reflected and transmitted probability densities.

2+2+4+2

4. The initial wave function for a simple harmonic oscillator is : 3+4+3

$$\psi(x,0) = \frac{2}{\sqrt{5}}\psi_1(x) + \frac{i}{\sqrt{5}}\psi_2(x)$$

where ψ_1 and ψ_2 are the normalized wave functions of the simple harmonic oscillator.

(i) Show that the wave function $\psi(x, 0)$ is normalized.

(ii) Calculate the expectation value of $\hat{H}$ in the state $\psi(x, 0)$.

(iii) Determine $\psi(x, t)$.

5. The eigen function for a Hamiltonian with a spherically symmetric potential $V(r)$ has the form :

$$\psi n_e m = R(r)\, Y_{e,m_e}(\theta, \phi)$$

where $Y_{e,m_e}(\theta, \phi)$ are the spherical harmonics :

(i) Write the eigen values of $\hat{L}^2$ and $\hat{L}_z$ for a spherical harmonic $Y_{e,m_e}(\theta, \phi)$.

(ii) Calculate the expectation value of the operator $\hat{L}_x^2 + \hat{L}_y^2$ for the state :

$$\psi = \frac{1}{5}[\psi_{211} + 3\psi_{210} + \sqrt{15}\,\psi_{21-1}]$$

(iii) Explain what is meant by the term centrifugal barrier. 2+5+3

6. (a) Show that the spectral representation of a Hermitian operator $\hat{A}$ with eigen values a_i and corresponding eigen vectors $|\phi i>$ is : 4

$$\hat{A} = \sum_{i=1}^{n} a_i \,|\phi i> <\phi_i|$$

(b) A vector space is spanned by the orthonormal eigenkets $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ and for any two state vectors : 6

$$|\psi_1\rangle = 2|\phi_1\rangle + i|\phi_2\rangle - i|\phi_3\rangle$$

$$|\psi_2\rangle = 2i|\phi_1\rangle - |\phi_2\rangle$$

(i) Calculate $\langle\psi_1|\psi_2\rangle, \langle\psi_2|\psi_2\rangle$ and $\langle\psi_1|\psi_2\rangle$.

(ii) Calculate the matrix elements for $|\psi_1\rangle\langle\psi_2|$.

7. (a) Define the ladder operators $\hat{a}$ and $\hat{a}^+$. Show that the simple harmonic oscillator Hamiltonian can be written as : 5

$$\hat{H} = \hbar\omega\left(\hat{a} + \hat{a} + \frac{1}{2}\right)$$

(b) Using the definition of $\hat{a}$ show that the ground state wave function of the simple harmonic oscillator is :

$$\psi_0(x) = N\exp\left(-\frac{a^2x^2}{2}\right)$$

where $a^2 = \frac{m\omega}{\hbar}$. 5

8. (a) Write down the angular momentum eigen states $|j, m_j\rangle$ for $j = 3/2$. What are the eigen values of $\hat{J}^2$ and $\hat{J}_z$ for each of these states ? What is the degeneracy of the state ? 5

(b) Using the definition :

$$\hat{S}_x = \frac{\hbar}{2}\left[|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\right]$$

for a spin half particle, show that the eigen vector $|S_{x\uparrow}\rangle$ is : 5

$$|S_{x\uparrow}\rangle = \frac{1}{2}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$

Physical Constants :

$h = 6.63 \times 10^{-34}$ Js

$\hbar = 1.05 \times 10^{-34}$ Js

$m_e = 9.11 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$ C