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M. Sc. (Physics) (MSCPH)

Term-End Examination

June, 2024

MPH-001 : MATHEMATICAL METHODS IN PHYSICS

Time : 2 Hours *Maximum Marks : 50*

Note : *Attempt any **five** questions. Marks are indicated against each question. Symbols have their usual meanings. You may use calculator.*

1. The expression for Bessel function of the first kind of order m is given by : 10

$$J_m(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{k!\,\Gamma(m+k+1)} \left(\frac{x}{2}\right)^{2k+m}$$

Using this relation, show that :

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi}}\, x^{-1/2} \sin x$$

and

$$J_{-1/2}(x) = \sqrt{\frac{2}{\pi}}\, x^{-1/2} \cos x$$

2. (a) The Helmholtz equation in Cartesian coordinates is :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f(x, y, z) + p^2 f(x, y, z) = 0 .$$

Reduce this equation into three ODEs. 5

(b) Using the generating function for Legendre polynomials : 5

$$g(x, t) = \frac{1}{\sqrt{1 - 2tx + t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n$$

obtain the recurrence relation

$$(2n+1) \times P_n(x) - (n+1) P_{n+1}(x) = n P_{n-1}(x)$$

3. (a) Show that the vectors : 5

$$\left\{ \vec{V}_1, \vec{V}_2 \right\}, \vec{V}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \vec{V}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

form a basis in R^2, the vector space of two-dimensional vectors.

(b) A rotation about origin by an angle ϕ of the orthogonal axes in a plane leads to the transformation of coordinates (of the same point) with respect to the two sets of axes by :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Show that this transformation matrix is orthogonal. Show that any 2 × 2 orthogonal matrix has this form. 5

4. For the real symmetric matrix : 2+5+3

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

obtain (i) eigenvalues, (ii) orthonormal eigen-vectors, and (iii) diagonalizing matrix.

5. Show that in polar form, the Cauchy-Riemann equations can be written as : 10

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

6. Using the method of residues, obtain the value of the following integral : 10

$$\int_0^{\infty} \frac{dx}{x^4 + 1}$$

7. (a) Using the method of Laplace transforms, solve the initial value problem : 7

$$y'' - 2y' + 5y = 0;\ y(0) = -1,\ y'(0) = 7$$

(b) Obtain the Fourier transform of the single square pulse represented by the function : 3

$$f(x) = \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

8. (a) Construct the multiplication table for the group of permutations of {1, 2, 3}. 5

(b) What is an Abelian group ? Explain with an example. 5