No. of Printed Pages : 5 **MST–018**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2024

MST-018 : MULTIVARIATE ANALYSIS

Time : 3 Hours *Maximum Marks : 50*

Note : (i) *Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions from the remaining question nos.* ***2*** *to* ***6****.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. State whether the following statements are True *or* False. Give reasons in support of your answers : 5×2=10

(a) The variance-covariance matrix of a random vector $\underset{\sim}{X}$ is symmetric.

(b) If $\underset{\sim}{X}$ is a p-variate normal vector and $a_{p\times1}$ and $b_{p\times1}$ are scalar vectors, then the linear combination $(\underset{\sim}{a}+\underset{\sim}{b})'\underset{\sim}{X}$ is a bivariate normal vector.

(c) The determinant of the matrix $\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1/6 \end{pmatrix}$ is 48.

(d) The first principal component is the normalized linear combination of random variables with minimum variance.

(e) Cluster analysis is a method of grouping a set of objects together, so that objects in the same group are not all similar.

2. (a) Let $\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ has the following joint density function : 5

$$f(x_1, x_2) = \begin{cases} 9x_1^2 x_2^2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & , \text{otherwise} \end{cases}$$

Find the mean vector and variance-covariance matrix of $\underset{\sim}{X}$. Also, comment on the independence of X_1 and X_2.

(b) Let $\underset{\sim}{X}_{P\times 1} \sim N_P(\underset{\sim}{\mu}, \Sigma)$. If $A_{P\times P}$ and $B_{P\times P}$ are two matrices of constant elements then obtain the variance of $A\underset{\sim}{X}$ and prove, that $A\underset{\sim}{X}$ and $B\underset{\sim}{X}$ are independent if $A\Sigma B'=0$, where 0 is the null matrix. 5

3. Let $\underset{\sim}{X}_{P\times 1} \sim N_P(\underset{\sim}{\mu}, \Sigma)$. Also, $\underset{\sim}{X}$, $\underset{\sim}{\mu}$ and Σ be partitioned as :

$$\underset{\sim}{X}_{P\times 1} = \begin{pmatrix} \underset{\sim}{X}^{(1)}_{K\times 1} \\ \underset{\sim}{X}^{(2)}_{(P\times K)\times 1} \end{pmatrix}, \quad \underset{\sim}{\mu}_{P\times 1} = \begin{pmatrix} \underset{\sim}{\mu}^{(1)}_{K\times 1} \\ \underset{\sim}{\mu}^{(2)}_{(P-K)\times 1} \end{pmatrix}$$

and

$$\Sigma_{P\times P} = \begin{pmatrix} \Sigma_{11_{K\times K}} & \Sigma_{12_{K\times (P-K)}} \\ \Sigma_{21_{(P-K)\times K}} & \Sigma_{22_{(P-K)\times (P-K)}} \end{pmatrix}$$

Then obtain the conditional distribution of $\underset{\sim}{X}^{(2)}$ given $\underset{\sim}{X}^{(1)} = \underset{\sim}{x}^{(1)}$.

Also, if $\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \end{pmatrix} \sim N_4(\underset{\sim}{\mu}, \Sigma)$, where $\mu = \begin{pmatrix} -4 \\ 1 \\ \hline 4 \\ 0 \end{pmatrix}$

and $\Sigma = \left(\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ \hline 1 & 2 & 6 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$, then obtain the conditional distribution of $\underset{\sim}{X}^{(2)}$ given $\underset{\sim}{X}^{(1)} = \underset{\sim}{x}^{(1)}$. 10

4. (a) If $\underset{\sim}{X} \sim N_4(\underset{\sim}{\mu}, \Sigma)$ with $\underset{\sim}{\mu} = \begin{pmatrix} 1 \\ 3 \\ 2 \\ -1 \end{pmatrix}$ and

$$\Sigma = \begin{pmatrix} 5 & 0 & 2 & 0 \\ 0 & 4 & 0 & -1 \\ 2 & 0 & 3 & 0 \\ 0 & -1 & 0 & 2 \end{pmatrix},$$ then prove that the sub-vectors $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ X_4 \end{pmatrix}$ are independent. 5

(b) Define Wishart matrix. Prove that if $A \sim w(n, P, \Sigma)$, then

$$Z_{q \times q} = CAC' \sim w(n, q, C\Sigma C'),$$

where C is a $q \times p$ matrix of rank $q (\leq p)$. 5

5. If :

$$A = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 4 & -2 \\ 0 & -2 & 6 \end{pmatrix},$$

then :

(i) obtain the square root matrix corresponding to a matrix A and verify that $A^{1/2}A^{1/2} = A$.

(ii) determine the first principal component of A and the proportion of the total variability that it explains. 5+5

6. (a) If :

$$\underset{\sim}{X}_{\alpha}^{(1)}, (\alpha = 1, 2, ..., N_1) \sim N_P(\underset{\sim}{\mu}^{(1)}, \Sigma)$$

and

$$\underset{\sim}{X}_{\alpha}^{(2)}, (\alpha = 1, 2, ..., N_2) \sim N_P(\underset{\sim}{\mu}^{(2)}, \Sigma)$$

are two independent random samples, where Σ is the common dispersion matrix and it is unknown, then describe the procedure for testing the hypothesis : 5

$$H_0 : \underset{\sim}{\mu}^{(1)} = \underset{\sim}{\mu}^{(2)}$$

(b) Define Mahalanobis D^2 and Hotelling's T^2. Also, state the relationship between the two. 5