No. of Printed Pages : 4 **MST-012**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

June, 2024

MST–012 : PROBABILITY AND PROBABILITY DISTRIBUTIONS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions from Question Nos.* ***2*** *to* ***6****.*

(iii) Use of scientific calculator (non-programmable) is allowed .

(iv) Symbols have their usual meanings.

1. State whether the following statement is *True* or *False*. Give reason in support of your answer :

5×2=10

(a) "If (Ω, F, P) be a probability space, then use assign probability to the members of Ω."

(b) If $X \sim \text{Bin}\left(10, \frac{1}{2}\right)$, then find P[X = 5].

(c) If $X \sim$ Unif (2, 22), then write CDF of X.

(d) Write definition of almost sure convergence and convergence in probability.

(e) Which one of the following inequalities is correct ?

$E(|X|) \geq |E(X)|$ or $E(|X|) \leq |E(X)|$

Give the proof in support of your answer.

2. (a) A pair of two fair tetrahedral dice is thrown. Sum of the two outcomes is noted in each throw. Find the probability that a 4 will be observed before observing a 5 in this random experiment. 6

(b) If Ω be the sample space of the random experiment of tossing a coin and F_1, F_2 be respectively the smallest and the largest σ-fields on Ω. Is the measure $\mu : F_1 \to [0, 1]$ defined by :

$$\mu(E) = \begin{cases} 0, & \text{if } E = \phi \\ 1, & \text{if } E \neq \phi \end{cases}, E \in F_1$$

a probability measure ? Also, check whether the same measure defined on the largest σ-field F_2, i.e., $\mu : F_2 \to [0, 1]$ defined by :

$$\mu(E) = \begin{cases} 0, & \text{if } E = \phi \\ 1, & \text{if } E \neq \phi \end{cases}, E \in F_4$$

is a probability measure.

3. (a) If $X = (X_1, X_2, X_3) \sim$ multinom $\left(20, \frac{1}{10}, \frac{3}{10}, \frac{6}{10}\right)$, then find mean vector of X. Also, find variance covariance matrix of the random vector X. 4

(b) If the probability that an individual suffers a bad reaction from an injection of a given serum is 0.001, using an appropriate probability distribution find the probability that out of 500 individuals :

(i) exactly 10,

(ii) more than 10,

individuals suffer from bad reaction. Assume that each individual has almost equal chance of a bad reaction. 6

4. (a) A biased coin which has the probability of getting a head as $\frac{1}{4}$ and a tail as $\frac{3}{4}$ is tossed till we get the first head. If X counts the number of failures before the first head, then obtain the PMF and CDF of X and also plot the PMF and CDF of X. 6

(b) Arnav and Abhishek usually play table tennis. On the basis of the past experience, it is known that the probability that Arnav beats Abhishek is 0.55. Assume that each game is independent of the other. One day they decide to play fill one of them wins five games. Find the probability that result of eighth game decide the winner. 4

5. State and prove central limit theorem. 10

6. (a) Suppose a coin is tossed infinitely many times. Let A_n, $n = 1, 2, 3, \ldots\ldots\ldots$ be a sequence of the independent events, where A_n represents the event of getting head in nth toss of the coin. Let P be a probability measure defined as follows : $P(A_n) = \frac{1}{n}$, $n = 1, 2, 3, \ldots\ldots$ Show that : 3

$$P\left(\{A_n \; i.o.\}\right) = 1.$$

(b) State Caratheodory's extension theorem. 1

(c) State four properties of CDF. Prove any *two* of them. 6