No. of Printed Pages : 5

MMTE–005

MASTER'S DEGREE PROGRAMME (MDP)
Term-End Examination
June, 2024

MMTE–005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) There are six questions in this paper.*

(ii) Question No. ***6*** *is compulsory.*

(iii) Do any ***four*** *questions from question nos.* ***2*** *to* ***5****.*

(iv) Use of calculator is not allowed.

1. (a) Check whether the [4, 2]-code : 3

$$C = \{(\bar{0},\bar{0},\bar{0},\bar{0},\bar{0},\bar{0},\bar{0})\ (\bar{1},\bar{1},\bar{1},\bar{0},\bar{0},\bar{0})\ (\bar{0},\bar{0},\bar{0},\bar{1},\bar{1},\bar{1})\ (\bar{1},\bar{1},\bar{1},\bar{1},\bar{1},\bar{1})\}$$

over $\mathbf{F}_2$ is perfect.

(b) Define a primitive element of a finite field. Check whether 5 is primitive element for the field $\mathbf{F}_{13}$. 3

(c) Let **C** be a [7, 4] binary cyclic code with generator polynomial $x^3 + x^2 + 1$. Find a generator matrix and a parity check matrix of the code. 4

2. (a) Define an [n, k]-linear code over a finite field $\mathbf{F}_q$. Check whether the code : 2

$$\mathbf{C} = \left\{(\bar{0},\bar{0},\bar{0}),(\bar{1},\bar{1},\bar{0}),(\bar{1},\bar{0},\bar{0}),(\bar{1},\bar{1},\bar{1})\right\}$$

over $\mathbf{F}_2$ is linear.

(b) What is a repetition code ? What is the generator matrix of the repetition code over $\mathbf{F}_2$ in which a message of length 3 is repeated twice ? If, in a repetition code in which a message of length three is sent thrice, the codeword 100 110 100 is received, decode the message assuming there is at most one error. 3

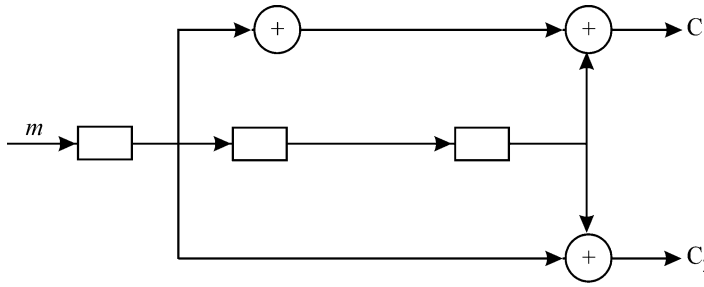

Figure 1 : Encoder for convolution code.

i	α^i		i	α^i
1	α		13	4α
2	$\alpha+3$		14	$4\alpha+2$
3	$4\alpha+3$		15	$\alpha+2$
4	$2\alpha+2$		16	$3\alpha+3$
5	$4\alpha+1$		17	$\alpha+4$
6	2		18	3
7	2α		19	3α
8	$2\alpha+1$		20	$3\alpha+4$
9	$3\alpha+1$		21	$2\alpha+4$
10	$4\alpha+4$		22	$\alpha+1$
11	$3\alpha+2$		23	$2\alpha+3$
12	4		24	1

Table 1 : Powers of $\alpha \in F_{25}$, where $\alpha^2 + 4\alpha + 2 = 0$.

(c) Define a cyclic code. Check whether the code {110, 011, 101} is cyclic. 2

(d) Find the gcd of $x^4 + x^3 + x + 2$ and $x^4 + 2x^3 + 2x + 2$ in $\mathbf{F}_3[x]$. 3

3. (a) Show that the $\mathbf{Z}_4$-linear codes with generator matrices : 5

$$G_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 2 \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 3 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}.$$

are monomically equivalent.

(b) Find the convolutional code for the message 11011. The convolutional encoder is given in figure 1. 5

4. (a) Construct a [12, 8] BCH code over $\mathbf{F}_5$ with designed distance 3. Use $x^2 + 4x + 2 \in \mathbf{F}_5[x]$ as the primitive polynomial and Table 1. 7

(b) If a polynomial generator matrix of an $[n, k]$ convolutional code $\mathbf{C}$ is basic and reduced, then prove that it is canonical. 3

5. (a) Find the weight distribution of the binary code generated by : 5

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Find the weight enumerator polyomial of the code. Also, find the weight enumerator polynomial of the dual code.

(b) The systematic generator matrix for a [5, 2] linear code is : 5

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Find the standard array for syndrome decoding.

6. Which of the following statements are true and which are false ? Justify your answer with short proof or a counter-example : 5×2=10

(a) $2 + x + x^2 + x^3$ is irreducible in $\mathbf{F}_3[x]$.

(b) If $GG^t = 0$ for the generator matrix G of a linear code, G is self dual.

(c) A quadratic residue code of length 7 exists over $\mathbf{F}_3$.

(d) The parity check code of a turbo code can be the identity matrix.

(e) Every perfect code is a self dual code.