No. of Printed Pages : 4

MMTE–001

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

June, 2024

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Answer any* ***four*** *questions from Question Nos.* ***2*** *to* ***7****. Use of calculators is not allowed.*

1. State whether the following statements are true *or* false. Justify your answers with a short proof or a counter-example : 10

   (i) If G and H are isomorphic graphs, then $\bar{G} \cong \bar{H}$.

   (ii) $K_{2,n}$ is planar for all n.

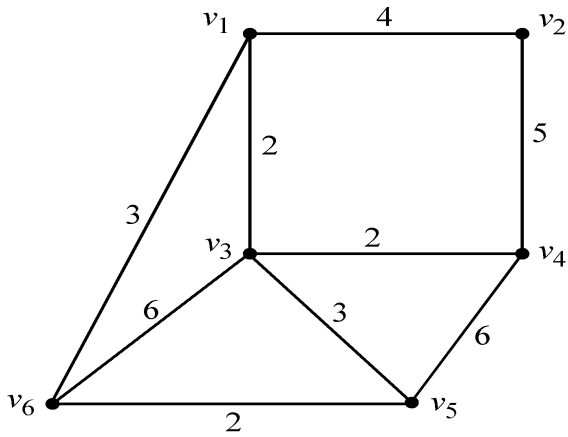
   (iii) (4, 3, 1, 1, 0) is a graphic sequence.

   (iv) L (K_1,n) is Hamiltonian for all $n \geq 3$.

(v) There exists a graph with K (G) = 1, K' (G) = 2 and $\delta(G) = 4$.

2. (a) Let v be a cut-vertex of a graph G. Prove that $\bar{G} - v$ is connected. 3

(b) Find a minimum weight spanning tree by using Kruskal's algorithm in the following weighted graph : 4



(c) What is $\chi'(K_{m,n})$? Justify your answer. 3

3. (a) Prove that every connected graph has at least one spanning tree. 3

(b) If M is a perfect matching in a graph G, then is it possible to have an M-augmenting path in the graph ? Justify your answer. 3

(c) Prove that a graph is 2-colorable iff it is a bipartite graph 4

4. (a) Prove that every graph $G \neq K_2$ has fewer cut-vertices than blocks. 5

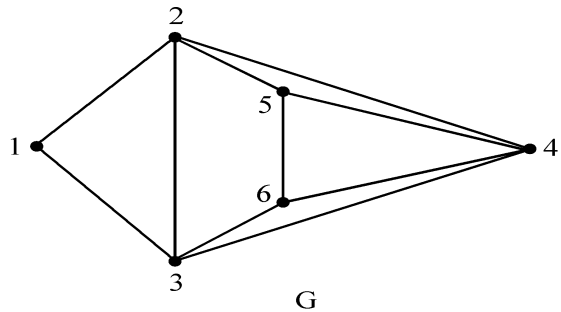
(b) Let G be a simple connected planar graph with n vertices, m edges and r faces, then prove that $n-m+r=2$. 5

5. (a) Prove that :

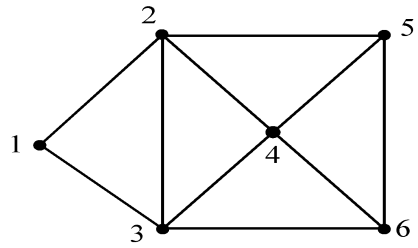
$$d=(7,6,3,3,2,2,2,1)$$

is a graphic sequence. 4

(b) The following diagrams show two different plane drawing of the same planar graph. Show that their duals are not isomorphic. 6

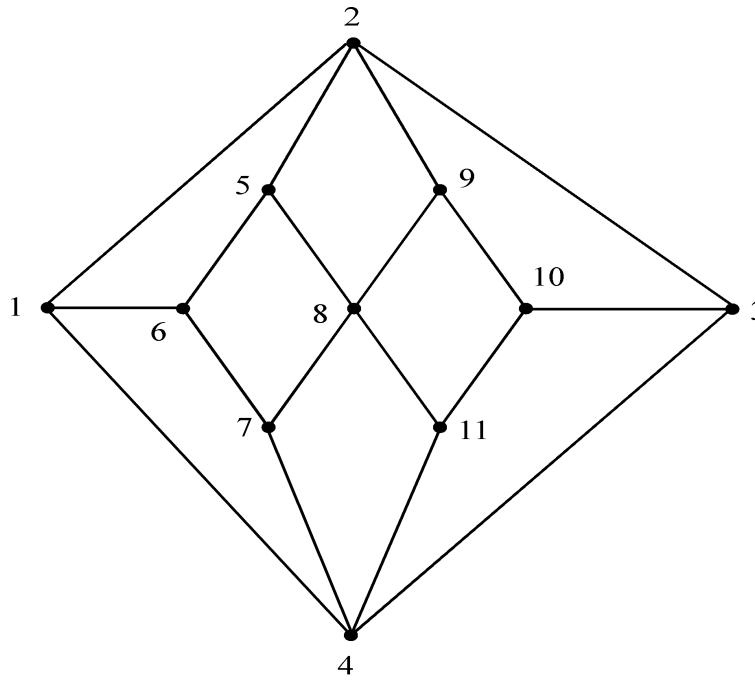


G

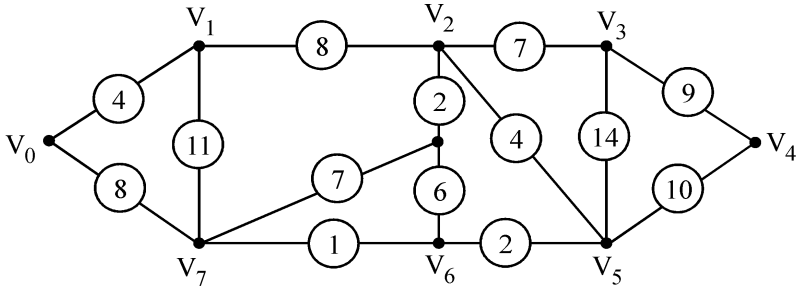


H

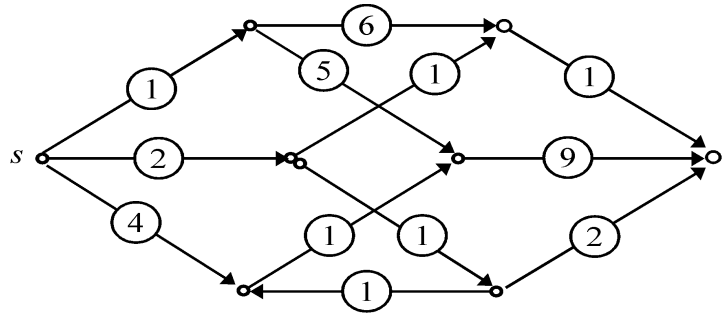
6. (a) Check whether the following graph is Hamiltonian or not :



(b) Using Dijsktra's algorithm, find a shortest path from vertex V_0 to vertex V_4 in the following weighted graph : 5



7. (a) Find the maximum flow value in the following network : 4



(b) Define vertex-colouring. Give an example of a 4-chromatic graph, with justification. 3

(c) For any graph G, show that $\beta(G) \leq \Delta(G)\alpha(G)$. 3