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M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

June, 2024

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ Hours *Maximum Marx : 25*

Note : *(i) Attempt any **five** questions.*

(ii) Use of scientific non-programmable calculator is allowed.

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1. (a) Consider the population dynamics to be governed by the equation :

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right) - P(N)$$

where r is the birth rate and k is the carrying capacity of the population. Find out the steady states and analyse the stability of the given system. 3

(b) Suppose that a population of yeast satisfying exponential growth model increases by 10% in an hour. If the initial population of yeast is 100,000 then find the population of yeast after three hours. 2

2. Patients arrive at the outpatient department of a hospital in accordance with a Poisson process at the mean rate of 12 per hour and the distribution of time for medical examination by an attending physician is exponential with a mean of 10 minutes. What is the minimum numbers of physicians to be posted for ensuring a steady-state distribution ? Calculate (i) the expected waiting time of a patient prior to being examined, (ii) the expected number of patients in the out-patient department, and (iii) the expected number of physicians remaining idle. Also, find the average time a patient has to spend in the out-patient department.

3. Use least-square method to fit a curve of the form $y = ae^{bx}$ to the data : 5

x	y
1	7.209
2	5.265
3	3.346
4	2.809
5	2.052
6	1.499

4. A company has four plants P_1, P_2, P_3, P_4 from which it supplies to three markets M_1, M_2, M_3. Determine the optimal transportation plan from the following data giving the plan to market shifting costs, quantities available at each plant and quantities required at each market : 5

Market ↓	P_1	P_2	P_3	P_4	Required at Market
M_1	19	14	23	11	11
M_2	15	16	12	21	13
M_3	30	25	16	39	19
Available at Plant	6	10	11	15	43

5. (a) Explain the following terms giving an example of each : 2

(i) Static and Dynamic models

(ii) Deterministic and stochastic models

(b) The return distribution on the two securities X and Y are given in the table below :

Possible Rates of Return		Associated Probability
X	Y	$P_{xj} = P_{yj}$
.19	.18	0.33
.17	.16	0.25
.11	.11	0.22
.10	.9	0.20

Find σ_{xy} and ρ_{xy}. 3

6. Do the stability analysis of one of the equilibrium solution of the following competing species system of equation with diffusion and advection :

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} - V_1 \frac{\partial N_1}{\partial x}$$

$$\frac{\partial N_2}{\partial t} = -d_1 N_2 + c_1 N_1 N_2 + D_2 \frac{\partial N_2}{\partial x^2} - V_2 \frac{\partial N_2}{\partial x},$$

$$0 \le x \le 2.$$

where V_1 and V_2 are constant advection velocities in x direction of the two populations with densities N_1 and N_2, respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, c_1 is the conversion rate. D_1 and D_2 are diffusion constant. The initial and boundary conditions are :

$$N_i(x,0) = f_i(x) > 0$$

$$0 \le x \le L, i = 1,2$$

$$N_i = \bar{N}_i \text{ at } x = 0 \text{ and } x = L \;\forall\; t, i = 1,2.$$

where $\bar{N}_i$ are the equilibrium solutions of the given system of equations. Also, write the limitation of this model. 5

7. (a) The control parameters of growth and decay of a tumour are respectively 1500 and 800 per day. Also, the damaged cells migrate due to vascularization of blood at a rate of 300 cells per day. Use logistic model to find the ratio of the growth of tumour after 20 days with the initial tumour. 3

(b) The deviation $g(t)$ of patient's blood glucose counteraction satisfies the differential equation :

$$3\frac{d^2g}{dt^2}+8\alpha\frac{dg}{dt}+27\alpha^2 g=0$$

for α being a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identity the type (overdamped, underdamped or critically damped) of this differential equation. Find the condition on α for which the patient is normal. 2