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MMT-007

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS [M.Sc. (MACS)]

Term-End Examination

June, 2024

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Note :– Question No. **1** is compulsory. Attempt any *four* questions from question No. 2 to 7. Use of scientific non-programmable calculators is allowed.

1. State whether the following statements are true or false. Justify your answer with the help of a short proof or a counter example : 2×5=10

P.T.O.

(i) The Lipschitz constant for the function $f(x, y) = x^2|y|$, defined on $|x| \leq 1, |y| \leq 1$ is equal to 1.

(ii) $L[t^2 \sin 5t] = \dfrac{3s^2 - 25}{(s^2+25)^3}$, where L is the Laplace transform.

(iii) The partial differential equation :

$$u_{xx} + 4x\,U_{xy} + (1 - y^2)\,u_{yy} = 0$$

is an elliptic p.d.e., inside the ellipse $4x^2 + y^2 < 1$.

(iv) The Runge-Kutta method of second order is nothing but the modified Euler's method.

(v) If H_n is a Hermite polynomial of degree n, then :

$$H_{2n}(O) = \frac{(-1)^n(2n+1)!}{(n+1)!}$$

2. (a) Find the fourier transform of $e^{-\pi x^2}$. 4

(b) Find the solution of the initial boundary value problem :

$$u_t = u_{xx},\ 0 \leq x \leq 1,\ t > 0$$

with conditions :

$$u(x,0)=\begin{cases} 2x & , \quad 0 \le x \le \frac{1}{2} \\ 2(1-x) & , \quad \frac{1}{2} < x \le 1 \end{cases}$$

$u(0, t) = 0 = u(1, t)$, using Crank-Nicholson method with $\lambda = \frac{1}{2}$. Assume $h = \frac{1}{4}$ and interpret for one time interval. 6

3. (a) If

$$f(x)=\begin{cases} 0 & , \quad -1 \le x \le 0 \\ x & , \quad 0 < x < 1 \end{cases},$$

then show that :

$$f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) - \frac{3}{52} P_4(x) + \ldots\ldots.$$

where $P_n(x)$ is the Legendre polynomial of degree n. 4

(b) Solve, in series, the differential equation : 6

$$x^2 y'' + 6xy'' + (6 + x^2)\, y = 0, \text{ about } x = 0$$

P.T.O.

4. (a) Given $\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$ and $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$, evaluate $y(0.4)$ using Milne's predictor-corrector method. Use one corrector iterations. 6

(b) Using second order finite difference method, solve the boundary value problem :

$$x^2 y'' = 2y - x, \; y(2) = 0 = y(3)$$

with $h = \frac{1}{3}$. 4

5. (a) Find the Fourier integral representation of the function : 4

$$f(x)=\begin{cases} 2 & , \quad |x| < a \\ 0 & , \quad |x| > a \end{cases}$$

(b) Find the Fourier inverse transform $F^{-1}\left[\frac{1}{\alpha^2 + 1}\right]$. 6

6. (a) For the boundary value problem $\frac{d^2y}{dx^2} = e^{x^2}, y(0) = 0, y(1) = 0$ estimate, using second order finite difference method, the values of $y(x)$ at x = 0.25, 0.5 and 0.75. (Given $e^{1/4} = 1.2840$, $e^{1/16} = 1.0645$, $e^{9/16} = 1.7551$). 5

(b) Solve the IVP $y' = x^2 + y^2$, $y(0) = 1$ upto $x = 0.4$ using fouth order Taylor series method with $h = 0.2$. 5

7. (a) Find the Fourier cosine transform of the function : 5

$$f(x) = \begin{cases} \cos x & , \quad 0 < x < a \\ 0 & , \quad x > a \end{cases}$$

(b) Solve : 5

$$x^3D^3y + 3x^2D^2y + xDy + y = x + ln\ x$$

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