MMT–005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCES) (MACS)

Term-End Examination

June, 2024

MMT-005 : COMPLEX ANALYSIS

Time : 1½ Hours   *Maximum Marks : 25*

Note : Question no. **1** is compulsory. Attempt any three questions from Questions No. **2 to 5**. Use of calculator is not allowed.

1. State giving reasons whether the following statements are True or False : 5 × 2 = 10

   (a) The set S = $\{z \in C : |z| < 1 \text{ or } |z - 2| < 1\}$ is connected.

   (b) There exists a nonconstant analytic function $f(z)$ in a domain D such that Im (f) is constant.

   (c) $f(z) = \dfrac{1}{\sin\left(\dfrac{1}{z}\right)}$ has a non-isolated singular point.

   (d) The radius of convergence of the power series
   $$\sum_{n=0}^{\infty}\left(\frac{n+1}{5n+3}\right)^{n}(z-2)^{n} \text{ is 2.}$$

   (e) The function $f(z) = z - \dfrac{1}{Z}$ is conformal at $z = \pm i$.

2. (a) Find the harmonic conjugate of the function
   $$u(x, y) = x^3 - 3xy^2. \qquad 2$$

   (b) Find the Laurent series expansion in powers of z for the function
   $$f(z) = \frac{1}{z^2 - 5z + 6}$$
   which is valid in the region $2 < |z| < 3$. 3

3. (a) Show that :
   $$\left|\int_C \frac{dz}{z^2 - 1}\right| \le \frac{3\pi}{16},$$
   Where C is the arc of the circle $|z| = 3$ lying in the first quadrant. 2

(b) Find, for the function   3

$$f(z) = \frac{\sin z^2 - z^2}{z^{10}}$$

(i)   The principle part at $z = 0$.

(ii) the nature of the singular point $z = 0$.

(iii) residue at $z = 0$.

4. (a) Show that a Mobius transformation which has three fixed points must be an identity map. 3

(b) Find all the solutions of the equations   2

$$e^z = -2.$$

5. Using the method of contour integration evaluate the improper integral.   5

$$\int_0^\infty \frac{dx}{(1+x^2)^2}$$

***