No. of Printed Pages : 4

MMT–004

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE [M.Sc. (MACS)]

Term-End Examination

June, 2024

MMT-004 : REAL ANALYSIS

Time : 2 Hours *Maximum Marks : 50*

Note : Question no. 1 is compulsory. Attempt any four questions from question no. 2 to 6. Calculators are not allowed. Notations as in the study material.

1. State whether the following statements are true or false. Justify your answers. $5 \times 2 = 10$

(a) The restriction of a continuous map defined on a metric space to any subset of the space is continuous.

(b) A finite subset of a metric space is totally bounded.

P.T.O.

(c) A connected subspace in a metric space which in not properly contained in any other connected subspace is always open.

(d) The surface given by the equation $x + y + z - \sin(xyz) = 0$ can also be described by an equation of the form $z = f(x, y)$ in a neighbourhood of the point (0, 0).

(e) A real valued function f on $[a, b]$ is continuous if it is integrable on $[a, b]$.

2. (a) Find the interior, closure, the set of limit points and the boundary of the set

$$A = \{(x, y) \in \mathbf{R}^2 : y = 1\}$$

in $\mathbf{R}^2$ with the standard metric. 3

(b) Consider $f : \mathbf{R}^3 \to \mathbf{R}^3$ given by 4

$$f(x, y, z) = (2x + 3y + z, xy, yz + xz)$$

Find $f'(2, 0, -1)$.

(c) Define the outer measure m^* of a subset A of $\mathbf{R}$. Find the outer measure of

$$A = \{x \in \mathbf{R} : x^2 = 1\} \cup [-3, 2]$$

3. (a) State Cantor's intersection theorem. Does the theorem hold for the metric space X = (0, 1] with the standard metric ? Justifiy your answer. 4

(b) Obtain the second Taylor's series expansion for the function given by $f(x, y) = xy^2 + 5xe^y$ at $\left(1, \frac{\pi}{2}\right)$. 3

(c) Define the Lebesgue integral of a simple measurable function. Find the integral of the function f given by

$$f = \chi_{[0, 1]} + 2\chi_{[3, 5]} + \chi_{[6, 9]}.$$

4. (a) Show that the continuous image of a compact metric space in compact. Is $f[0, 1]$ compact in **R** where $f(x) = e^x$. 4

(b) Check whether the function $f : \mathbf{R}^4 \to \mathbf{R}^4$ given by $f(x, y, z, w) = (x, x - y, y^2z, zw)$ is locally invertible at the point $(1, -2, 1, 0)$. 3

(c) State Fatou's Lemma. Using this Lemma prove the dominated convergence theorem.

5. (a) Show that the projection map $p : \mathbf{R}^2 \to \mathbf{R}$ given by $p(x, y) = y$ is uniformly continuous. 3

(b) Find and classify the extreme values of $(x, y) = xy$ 4

Subject to the constraint

$$\frac{x^2}{8} + \frac{y^2}{2} - 1 = 0.$$

(c) Prove that the Fourier series for $f(t) = t^2$ on $[-\pi, \pi]$ is

$$\frac{\pi^2}{3} + 4\sum_{x \in N} \frac{(-1)^n \cos nt}{n^2}$$ 3

6. (a) What are the components of **R** under : 2

(i) Standard metric on **R**.

(ii) Discrete metric on **R**.

(b) Define a stable system. Check whether the system

$\mathbf{R} : S \to S$ given by

$$g(t) = (\mathbf{R}f)(t) = \int_{-\infty}^{t} f(\tau) e^{-(t-\tau)d\tau}$$

is stable a not, where S denotes the set of signals. 3

(c) For $f, g, \in L'(R)$ define the convolution map $f * g$.

Show that if either f or g is bounded, then the convolution $f * g$ exists for all x in **R** and is bounded in **R**.

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