No. of Printed Pages : 3 **MMT-002**

MASTER IN MATHEMATICS WITH APPLICATIONS TO COMPUTER SCIENCE

Term-End Examination

June, 2024

MMT-002 : LINEAR ALGEBRA

Time : 1½ Hours *Maximum Marks : 25*

Note : (i) There are **five** questions in this paper.

(ii) The fifth question is compulsory.

(iii) Do any *three* questions from Q. 1 to Q. 4.

(iv) Use of calculators is *not* allowed.

1. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by :

$$T(x, y) = (x + y, x - y, -y)$$

P.T.O.

Find the matrix of T relative to the bases $\{(1, 1), (1, -1)\}$ of $\mathbb{R}^2$ and $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ of $\mathbb{R}^3$. 2

(b) Find the spectral decomposition of : 3

$$\begin{bmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

2. Solve the system of differential equations :

$$\frac{dy(t)}{dt} = Ay(t)$$

where $A = \begin{bmatrix} 0 & 4 \\ -1 & 4 \end{bmatrix}$, $y(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. 5

3. Find the singular value decomposition of :

$$A = \begin{bmatrix} 2 & -3 & 0 \\ -3 & -2 & 2 \end{bmatrix}$$

Also, find its Moore-Penrose inverse. 5

4. (a) If $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ determine the behaviour of A^i is $i \to \infty$. 2

(b) Find the QR decomposition of :

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$ 3

5. Which of the following statements are true and which are false ? Justify your answer with a short proof or a counter example, whichever is appropriate. 10

(i) If two matrices have the same characteristic polynomial and the same minimal polynomial they are similar.

(ii) Every positive definite matrix is invertible.

(iii) QR decomposition of any matrix is unique.

(iv) If $A \in M_n(\mathbb{C})$ A and A^* commute, then A is unitarily diagonalisable.

(v) If $A \in M_n(\mathbb{R})$, $A = N + D$, where N is a nilpotent matrix and D is a diagonal matrix, then $ND = DN$.

***