No. of Printed Pages : 4 **MMT–003**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M.Sc. (MACS)]

Term-End Examination

June, 2024

MMT-003 : ALGEBRA

Time : 2 Hours *Maximum Marks : 50*

Note : Q. 1 is compulsory. Answer any 4 questions from Q. 2 to 6. Calculator is not allowed.

1. State whether True or False giving reasons for your answer :

   (a) A group of order 56 must have a normal sylow subgroup. 2

   (b) It is not possible to find a field of positive characteristic all of whose finite extensions are separable. 2

   (c) All non-zero prime ideals in R[X] are maximal. 2

P.T.O.

   (d) In the ring of Gaussian integers $\mathbb{Z} + \mathbb{Z}i$, α is an irreducible element. 2

   (e) If m_1 and m_2 are natural numbers then there is always an element of order $m_1 m_2$ in $S_{m_1} + S_{m_2}$. 2

2. (a) Determine the maximum possible order of an element in S_{12}. 3

   (b) Let K be a Galois extension of a field F with Galois group G (K/F) = D_{10}, the dihedral group of order 10. How many fields L are there such that $F \subset L \subset K$? How many such L will be normal over F ? 4

   (c) How many proper ideals does the ring $\mathbb{Z}_{24}$ (under addition and multiplication modulo 24) have ? How many of these are maximal ? Explain your answers. 3

3. (a) Compute 247^{66} (mod 120). 3

   (b) Suppose a dihedral group D_{10} acts on a set with 13 elements and the action is without singleton orbits. Show that there must be a single orbit with 5 elements. 4

(c) Show that the subring $\mathbb{Z} + \mathbb{Z}\sqrt{-7}$ of the complex numbers cannot be a Euclidean domain. 3

4. (a) Compute the Legendre symbol $\left(\frac{77}{43}\right)$. 2

(b) Write down all the abelian groups (upto isomorphism) of order 280. Determine the invariants for each such group. 4

(c) Determine the splitting field K of the polynomial $X^8 - Z \in Q[X]$ over Q. Determine the cycles group G(K/Q). 4

5. (a) Let G be a group of odd order which has a subgroup H of index 3. Define an action of G on G/H. What will be the kernel of the resulting homomorphism from G to the permutation group on G/H ? Show that H must be normal in G. 3

(b) Write down 3 monic irreducible polynomials of degree 2 in $\mathbb{Z}_5[X]$. Justify the irreducibility. 3

(c) Is the quotient ring $\frac{\mathbb{Z}_7[X]}{< X^3 + X^2 - 3X - 3 >}$ an integral domain ? Why ? Write the ring in a simpler form. 4

6. (a) State the orbit counting theorem for the action of a finite group on a finite set. Explain your notation. 2

(b) What do you mean by a free group generated by a set S. Derive a reduced word from the word $abb^{-1}\, a^{-1}\, bab^{-1}$ with $S = \{a, b\}$. 3

(c) Use the Gram-Schmidt orthogonalization process to get an orthonormal basis for the subspace of $\mathbf{R}^3$ generated by the basis (1, 1, 0) and (2, – 1, 0). 3

(d) Show that $SO_2(\mathbf{R})$ is isomorphic to the unit circle (S',). 2

***