No. of Printed Pages : 7 **MSTE-002**

# POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

## Term-End Examination

## June, 2023

### MSTE-002 : INDUSTRIAL STATISTICS—II

*Time : 3 Hours* *Maximum Marks : 50*

***Note :*** (*i*) *Question No.* ***1*** *is compulsory.*

(*ii*) *Attempt any* ***four*** *questions from the remaining question nos.* ***2*** *to* ***7****.*

(*iii*) *Use of scientific calculator (non-programmable) is allowed.*

(*iv*) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*

(*v*) *Symbols have their usual meanings.*

1. State whether the following statements are True *or* False. Give reasons in support of your answers : $5 \times 2 = 10$

   (a) Suppose in a regression analysis, your variables are stored in different columns and observations are stored in rows. Then total sum of squares is independent of number of rows.

(b) If in a multiple linear regression analysis, there are four independent variables, where two of them are continuous, other two are categorical variables having 2 and 4 categories. Then total no. of regression equations will be 6.

(c) For a second order Autoregressive AR (2) process $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + a_t$ to be stationary, the following must hold : $\alpha_1 + \alpha_2 > 1, \alpha_2 - \alpha_1 < 1, -1 < \alpha_2 < 1$.

(d) For the given equations :

$$2x_1 + x_2 + 3x_3 + 4x_4 = 5$$

$$5x_1 + 3x_2 + 5x_3 + 5x_4 = 8$$

one of the basic solutions is

$$x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1.$$

(e) In a general queue denoted by :

$$A \mid B \mid c \mid K$$

K represents number of servers and $c$ represents buffer size.

2. (a) Consider a telephone booth with Poisson arrivals spaced 10 minutes apart on an

average, and exponential call lengths averaging 3 minutes. 5

(i) What is the probability that an arrival will have to wait more than 10 minutes before the phone is free ?

(ii) What is the probability that it will take him more than 10 minutes altogether to wait for phone and complete his call ?

(iii) Estimate the fraction of a day that the phone will be in use.

(iv) Find the average number of units in the system.

(b) A manufacturer has to supply his customer with 24000 units of his product every year. The demand is fixed and known. Since the unit is used by the customer in an assembly operation and the customer has no storage space for units, if the manufacturer fails to supply the required units, the shortage cost is ₹ 2 per unit per month. The inventory cost is ₹ 1 per unit per month and the set-up cost per run is ₹ 3,500. Determine : 5

(i) The optimal run size;

(ii) The optimal scheduling period; and

(iii) The minimum inventory cost.

3. (a) Solve the LPP :

Maximize :

$$Z = 6x_1 + 4x_2$$

Subject to the constraints :

$$2x_1 + 3x_2 \le 30$$

$$3x_1 + 2x_2 \le 24$$

$$x_1 + x_2 \ge 3$$

$$x_1, x_2, \ge 0.$$

using simplex method. Also find general solution. 7

(b) Show that the set :

$$S = \{(x, y) : 0 \le y \le 8, 0 \le x \le 3\}$$

$$\cup \{(x, y) : 3 \le x \le 9, 3 \le y \le 8\}$$

is not a convex set. 3

4. (a) Explain artificial variable technique and degeneracy in LPP. 4

(b) There are five jobs, each of which must go through two machines A and B in the order AB. Processing times are given as follows :

| Job | Processing time (hours) | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Time for A | 5 | 1 | 9 | 3 | 10 |
| Time for B | 2 | 6 | 7 | 8 | 4 |

Determine a sequence for five jobs that will minimise the elapsed time T. Calculate the total idle time for the machines. 4

(c) Differentiate between feasible solution and basic feasible solution. 2

5. (a) Consider the model

$$(1-0.5\,\mathrm{B})(1-\mathrm{B})\mathrm{X}_t = (1-0.2\,\mathrm{B})a_t$$

Express this model as ARIMA $(p, d, q)$ and write values of $p, d, q$. Also determine whether the process is stationary and invertible. 3

(b) Data on bill amount and tip amount of a restaurant are given as follows :

| **Bill Amount** | **Tip Amount** |
|---|---|
| 34 | 5 |
| 108 | 17 |
| 64 | 11 |
| 88 | 8 |
| 99 | 14 |
| 51 | 5 |

Identify independent and dependent variables. From a simple linear regression point of view, suppose as usual $x_i, y_i$ denote observed values of independent and dependent variables, respectively. Then obtain the three quantities

$$\sum_i \left(y_i - \hat{y}_i\right)^2, \sum_i \left(y_i - \bar{y}\right)^2, \sum_i (\hat{y}_i - \bar{y})^2$$

and interpret these quantities in words. 7

6. The data of 10 trips were recorded to collect information on :

(i) total km travelled

(ii) number of deliveries

(iii) the daily gas price, and

(iv) total travel time in hours

The data thus collected are given as follows :

| **km travelled ($x_1$)** | **No. of deliveries ($x_2$)** | **Gas prices ($x_3$)** | **Travel time ($y$) (hours)** |
|---|---|---|---|
| 89 | 4 | 3.84 | 7 |
| 66 | 1 | 3.19 | 5.4 |
| 78 | 3 | 3.78 | 6.6 |
| 111 | 6 | 3.89 | 7.4 |
| 44 | 1 | 3.57 | 4.8 |
| 77 | 3 | 3.57 | 6.4 |
| 80 | 3 | 3.03 | 7 |
| 66 | 2 | 3.51 | 5.6 |
| 109 | 5 | 3.54 | 7.3 |
| 76 | 3 | 3.25 | 6.4 |

Estimate the value of $y$ for $x_1 = 100$, $x_2 = 3$, $x_3 = 3.5$. 10

7. Explain the following in detail : 3+3+4

(i) Moving Average (MA) Process

(ii) Autoregressive (AR) Process

(iii) Autoregressive Moving Average (ARMA) Model