POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination
June, 2023
MST-003 : PROBABILITY THEORY

Time : 3 Hours   *Maximum Marks : 50*

Note : (i) *Question No.* **1** *is compulsory.*

(ii) *Attempt any* **four** *questions from the remaining Question Nos.* **2** *to* **7**.

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

1. State whether the following statements are True *or* False. Give reasons in support of your answers : 2 each

   (a) If a card is drawn from a well shuffled pack of 52 cards, then the probability that it is either club or king is $\frac{9}{26}$.

P. T. O.

(b) If a random variable X follows the probability distribution $p(x) = C \cdot \dfrac{1}{2^x}$, where $x = 1, 2, ...$, then the value of C is $\dfrac{1}{2}$.

(c) If 9 P[X = 4] = P[X = 2] for a binomial variate X having $n = 6$, then $P = \dfrac{1}{6}$.

(d) If $X \sim N(7, 12)$ and $Y \sim N(4, 9)$ are independent normal variates, then $(X - Y) \sim N(11, 3)$.

(e) If the probability that a student will get grade 'A' in a statistics course is 0.32, then the probability that he will get either grade 'A' or grade 'B' is 0.27.

2. (a) The odds that a book will be favourably reviewed by 3 independent critics are 5 to 2, 4 to 3 and 3 to 4, respectively. Find the probability that of the three reviews, a majority will be favourable.   5

(b) The numbers on a roulette wheel are 0, 1, 2, ..., 36 and 00. The numbers 0 and 00 are green, while the other even numbers are red and the odd numbers are black. Assuming that the wheel is fair, find the probability that the outcome is (i) 13 or green, (ii) greater than 17 and black, (iii) red or black. 　　　　　　　　　　3

(c) Explain axiomatic approach of probability theory. 　　　　　　　　　　　　2

3. (a) A random variable X has the following probability mass function :

Value of X = x	P (X = x)
0	a
1	$3a$
2	$5a$
3	$7a$
4	$9a$
5	$11a$
6	$13a$
7	$15a$
8	$17a$

(i) Determine the value of a.

P. T. O.

(ii) Find P(X < 3), P(X ≥ 3) and P(0 < X < 5).

(iii) What is the smallest value of x for which P(X ≤ x) > 0.5 ?   5

(b) The distribution function of a variable X is as follows :

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \dfrac{x}{8} & \text{if } 0 \le x < 2 \\ \dfrac{x^2}{16} & \text{if } 2 \le x < 4 \\ 1 & \text{if } x \ge 4 \end{cases}$$

Find E(X) and variance (X).   3

(c) Define random variable and write at least two advantages of a random variable.   2

4. (a) A taxi cab company has 12 Ambassadors and 8 Fiats. If 5 of these taxi cabs are in the workshop for repairs and an Ambassador is as likely to be in for repairs as a Fiat, what is the probability that (i) 3 of them are Ambassadors and 2 are Fiats, (ii) at least 3 of them are Ambassadors, and (iii) all 5 are of the same make ?   5

(b) An irregular 6-faced die is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets ?   5

5. (a) Buses arrive at a specified stop at 15 minute intervals starting at 7 am, that is, they arrive at 7:00, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a random time, that is uniformally distributed between 7:00 am and 7:30 am, find the probability that he waits (i) less than 5 minutes, and (ii) at least 12 minutes for a bus.   3

(b) The percentage X of a particular compound contained in a rocket fuel follows the distribution N (33, 9), though the specification for X is that it should lie between 30 and 35. The manufacturer will get a net profit (per unit of the fuel) of

₹ 100, if 30 < X < 35, of ₹ 50, if 25 < X ≤ 30 or 35 ≤ X < 40 and incur a loss of ₹ 60 per unit of fuel otherwise. Find the expected profit of the manufacturer.   3

(c) Give *four* applications of Poisson distribution with suitable example.   4

6. (a) A can hit a target in 4 out of 5 shots and B can hit the target in 3 out of 4 shots. Find the probability that (i) the target being hit when both try (ii) the target being hit by exactly one person.   2

(b) The distribution function F of a continuous variable is given by :

$$F(x) = 0 \text{ if } X < 0$$

$$= x^2, \text{if } 0 \le x \le \frac{1}{2}$$

$$= 1 - \frac{3(3-x)^2}{25}, \text{if } \frac{1}{2} \le x < 3$$

$$= 1, \text{if } x \ge 3.$$

Find the pdf of X and evaluate $P(|X| \le 1)$ and $P\left(\frac{1}{3} \le X < 4\right)$, using both F and f.   5

(c) Write three applications of exponential distribution.   3

7. (a) A manufacturer, who produces bolts, find that 0.1% of the bolts are defective. The bolts are packed in boxes containing 500 bolts. A service centre buys 100 boxes from the producer. Find the expected number of boxes which will contain at least two defective bolts.   4

(b) If the time (in hours) required to repair a machine is exponentially distributed with $\lambda = \frac{1}{2}$, then :

(i) What is the probability that the repair time exceeds 2 hours ?

(ii) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours ?

4

(c) Write at least *two* applications of beta distribution.   2