No. of Printed Pages : 4

MMTE-001

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2023

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is* ***compulsory****. Answer any* ***four*** *questions from Question Nos.* ***2*** *to* ***7****. Use of calculators is not allowed. Symbols used have their usual meaning.*

1. State whether the following statements are true or false. Justify your answers with a short proof or a counter-example : 5×2=10

   (i) If G is a graph with 100 vertices and 1500 edges, then $\bar{G}$ has 3450 edges.

   (ii) The radius and diameter of a complete bipartite graph are always equal.

   (iii) Q_3 is Eulerian.

   (iv) $K_{5,6}$ is 5-edge-chromatic.

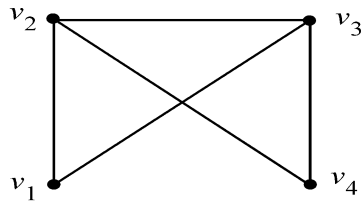
   (v) If a graph G has a perfect matching, then every maximal matching of G is perfect.

P. T. O.

2. (a) Does there exist a graph with all vertices having distinct degrees ? Justify your answer. 3

   (b) Show that for any connected graph G, diam (G) ≤ 2 rad (G). 3

   (c) Find the number of (v_1, v_2)-walks of length 3 in the following graph : 4

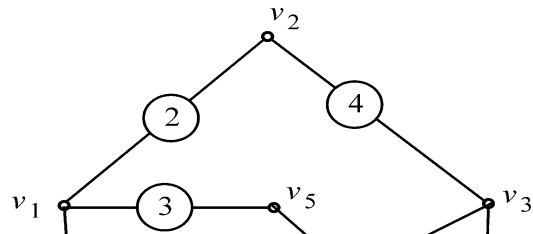


3. (a) Let G be a graph with exactly one path between every pair of vertices. Show that G is a tree. 3

   (b) Prove that a connected multigraph is Eulerian iff all of its vertices have even degree. 7

4. (a) Show that in a complete binary tree the number of leaves is exactly one more than the number of non-leaves. 4

   (b) Compute the distances of all the vertices from v_1 in the following graph, using Dijkstra's algorithm : 6

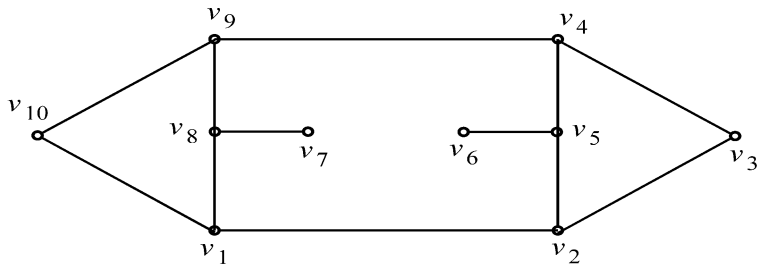


5. (a) Show that the Grötzsch graph is 4-chromatic. 5

(b) If G is a planar graph, then show that $\delta(G) \leq 5$. 3

(c) What is the thickness of the Peterson graph ? Justify your answer. 2

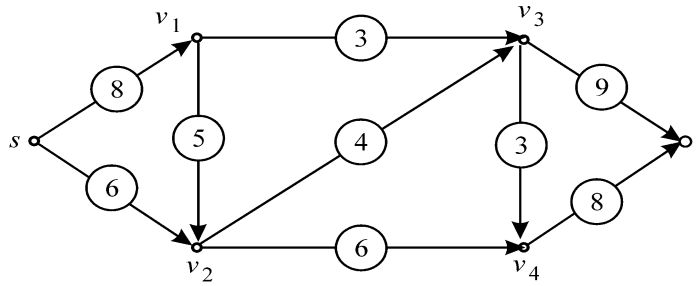
6. (a) Verify the König-Egerváry theorem for the following graph : 5



connected ? Justify your answer. 2

(c) Show that every Hamiltonian graph is 2-connected. 3

7. (a) Consider the following network N. 5



Define a function f on the edges set of N as follows :

$$f(sv_1) = 7,\ f(sv_2) = 3,\ f(v_1v_2) = 5$$

$$f(v_1v_3) = 2,\ f(v_2v_3) = 4,\ f(v_2v_4) = 4$$

$$f(v_3v_4) = 2,\ f(v_3t) = 6,\ f(v_4t) = 6$$

Check whether f represents a flow on N or not.

(b) Check whether the sequence : 5

(6, 6, 5, 5, 3, 3, 3, 3, 1, 1)

is graphic or not.