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MMT-009

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2023

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ *Hours* *Maximum Marks : 25*

Weightage : 70%

Note : *(i) Attempt any* ***five*** *questions.*

(ii) Use of calculator is not allowed.

1. (a) Find a linear demand equation that best fits the following data, and use it to predict annual sale of cars priced at ₹ 8,00,000 : 3

x = Price (lakh of ₹)	y = sale of new cars
12	120
10	132
18	90
20	65
14	100
25	20

(b) State the types of modelling you will choose for the following problems, giving reasons in support of your answer : 2

(i) Formation of sand and their encroachment its deforested lands near deserts

(ii) Advertising by a manufacturer to promote the product, improve sales and the revenue is generated.

2. Discuss the stability analysis of the following model governing action and diffusion of both-prey and predator populations : 5

$$\left.\begin{aligned} \frac{\partial N_1}{\partial t} &= a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} \\ \frac{\partial N_2}{\partial t} &= -d_1 N_2 + G N_1 N_2 + D_1 \frac{\partial^2 N_2}{\partial x^2} \end{aligned}\right\} \text{where } 0 \leq x \leq L$$

under the following initial coditions and no flux boundary conditions.

$$\left.\begin{aligned} N_1(x,0) &= f_1(x) > 0 \\ N_2(x,0) &= f_2(x) > 0 \end{aligned}\right\} \text{ for } 0 \leq x \leq L$$

$$\frac{\partial N_1}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = L \,\forall\, t$$

$$\frac{\partial N_2}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = L \,\forall\, t$$

The variables and parameters of the system given by above equations are as follows :

N_1 = density of prey population

N_2 = density of predator population

a_1 = growth rate
d_1 = death rate
b_1 = predation rate
c_1 = conversion rate
D_1, D_2 = diffusion cofficients

and a_1, d_1, b_1, c_1, D_1 and D_2 are all positive constants.

3. (a) The reproduction function of the cancer cells within a spherical tumour is given by : 3

$$\phi(c) = \frac{2c+1}{(1-3c)^2}; c \neq \frac{1}{3}$$

with initial condition $c = c_0$ at $t = 0$. Find the density of cancer cells in the tumour's surface area at $t = 20$ days.

(b) Calculate the expected return and risk of a security for the following information : 2

Probabilities	Possible returns
0.10	0.20
0.20	0.16
0.30	0.05
0.15	0.30
0.25	0.10

4. (a) H. P. computers has been amended a contract for installing computers in a college. The company has to make a choice between two alternatives : 3

(i) hire one or more computer technician for 8 hours a day

Or

(ii) hire one or more part time computer technician for 4 hours a day.

The rate of wages of computer technician is ₹ 40 per hour while the corresponding rate of part time technician is ₹ 16 per hour. The company wants to engage technicians for work not more than 125 man hours per day and limit the charges to techniques to ₹ 1,800. The company estimates that the productivity of a full time technician is eight units and a part-time apprentice technician is three units.

Formulate the integer programming problem to enable the company to select the optimum number of technicians and apprentices.

(b) The control parameters of growth and decay of a turn are respectively 2000 and 1200 per day. Also damaged cells migrate due to visualization of blood at a rate of 500 cells per day. Find the ratio of growth tumour after 30 days with initial tumour. 2

5. A goldsmith has three counters in its office. Customers are found to arrive, in a Poisson fashion, at an average rate of 20 per 8-hours day. The amount of time that a counter incharge takes with a customer is found to have exponential distribution with mean service time 30 minutes. Customers are processed in order of their appearance. 5

   (i) How many hours a week can a counter-incharge expect to spend with customers ?

   (ii) How much time, on the average, does a customer spend in the goldsmith's office.

6. An engineer obtained the following data between a vacuum setting and particle size distribution for a product : 5

Vacuum setting = x	Particle size = y
20	5
22	4
24	6
18	3
26	7
15	2

Fit a linear regression model to the above data.