No. of Printed Pages : 4 **MMT-003**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2023

MMT-003 : ALGEBRA

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Attempt any* ***four*** *questions from Question* ***No. 2*** *to* ***6****. Calculators are* ***not*** *allowed. Show all the steps involved. Do your rough work at the bottom or at the side.*

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1. Which of the following statements are true and which are false ? Give reasons for your answers :

5×2=10

(i) A_6 has 6 distinct normal subgroups.

(ii) Every finite extension of $K = Q\left(2^{\frac{1}{3}}, i\right)$ is separable over K.

(iii) $f : GL_1(\mathbf{R}) \to GL_1(\mathbf{R}) : f(x) = 2x$ is a linear representation.

(iv) If I and J are ideals of a commutative ring R, then I + J = R.

(v) U(K[x]) = K, where K is a field.

2. (a) Is 80 a square modulo 73 ? Give reasons for your answer. 2

(b) Define a normal subgroup of a group. Give an example, with justification, of a normal subgroup. 2

(c) Define the nil radical of a ring. Find the radical $\mathbf{Z}_9$. 2

(d) Define the characteristic of a field. Is it possible to have a field of characteristic 3 with more than 3 elements ? Justify your answer. 2

(e) Define a special linear group over $\mathbf{R}$. Give a non-trivial element of such a group, with justification. 2

3. (a) Let G be a group of order 21 and let G act on a set S, which has 23 elements. Suppose G does not fix any element of S. What are the possible cardinalities of the orbits under the action of G on S ? Justify your answer. 5

(b) Define the splitting field of a polynomial over a field. Let K be the splitting field of $x^4 - 7 \in \mathbf{Q}[x]$. Determine [K : **Q**]. 5

4. (a) Show that (**N**, *) natural numbers under multiplication is not a free semi-group. 3

(b) Give an example, with justification, of a cubic polynomial over **Q** with the degree of its splitting field over **Q** being less than 3.

2

(c) Apply the extended g.c.d. algorithm to write the g.c.d. of 17 and 7 as an integer linear combination of 17 and 7. 3

(d) Define the content of a polynomial over a PID. Give an example, with justification, of a quadratic polynomial over **Z** whose content is 2. 2

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5. (a) List all the non-isomorphic classes of abelian groups of order 900. 4

(b) Let K and L be finite extensions of $\mathbf{Q}$ and $\mathbf{C}$ such that $[K : \mathbf{Q}] = m$, $[L : \mathbf{Q}] = n$ and $(m, n) = 1$. Show that $[LK : \mathbf{Q}] = mn$. 4

(c) Let G be a group and $N \Delta G$. Let $x \in N$. Find $C_x \setminus N$. 2

6. (a) Check whether or not a group of order 70 is simple. 3

(b) Describe the special orthogonal group in GL (2, $\mathbf{R}$). Show that it is isomorphic to the unit circle (group under multiplication). Also show that this group is conjugate to a diagonal group in GL (2, $\mathbf{C}$). 7