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MMT-002

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2023

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Weightage : 70%

Note : *Question No.* ***5*** *is compulsory. Answer any* ***three*** *questions from Q. Nos.* ***1*** *to* ***4****.* *Calculators are not allowed.*

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1. (a) Find a QR decomposition of the matrix : 2

$$\begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}.$$

(b) Let the matrix of a linear operator T with respect to an ordered basis $B_1 = \{u_1, u_2, u_3\}$ of $\mathbf{R}^3$ be : 3

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrix of T with respect to the basis :

$$B_2 = \{u_1 + u_2, u_2 + u_3, u_1 + u_2 + u_3\}$$

2. (a) Check whether the system $Ax = y$ is consistent, where : 3

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \text{ and } y = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 3 \end{bmatrix}.$$

(b) Write all possible Jordan canonical forms for a 5×5 matrix whose minimal polynomial is $(x-1)(x-2)(x-3)$ and the determinant is 12. 2

3. (a) Write a unitary matrix whose first column is $\frac{1}{\sqrt{3}}\begin{bmatrix} i \\ i \\ 1 \end{bmatrix}$. 2

(b) Evaluate e^{A}, where : 3

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}.$$

4. (a) Write the spectral decomposition of the matrix : 3

$$\begin{bmatrix} 1 & 5 & -3 \\ 0 & 5 & -2 \\ 0 & 3 & 0 \end{bmatrix}.$$

(b) Check whether the matrix : 2

$$A = \begin{bmatrix} 0 & -3 & 2 \\ 0 & 2 & -1 \\ 0 & 4 & -2 \end{bmatrix}$$

is nilpotent.

5. Which of the following statements are true and which are not ? Give reasons for your answers : 2×5=10

(i) If a linear operator T on a finite dimensional vector space is not one-one, then T has an eigen value zero.

(ii) The QR-decomposition of any non-singular matrix is unique.

(iii) There is a unitary matrix with eigen values 2 and $\frac{1}{2}$.

(iv) If the eigen values of $A \in M_2(\mathbb{C})$ are 3, 2, $\det(e^{A}) = e^{t}$.

(v) If $A \in M_n(\mathbb{C})$ such that $t_r(AA^*) = 0$, then $A = 0$.